Comment on “Quantum State Transfer Between Matter and Light”

Matsukevich and Kuzmich (1) describe an interesting experiment directed toward the realization of scalable quantum networks through the protocol of (2). In particular, they demonstrate coherence between two atomic ensembles comprising two cylindrical volumes of cold rubidium atoms within a single magneto-optical trap. The authors claim that their measurements “constitute a complete set of tools required to build an arbitrary large-scale quantum network” and assert that this advance is achieved by way of several essential steps involving quantum entanglement.

Certainly, entanglement is a crucial requirement for the quantum repeater architecture proposed in (2). It is important to realize, however, that every measurement presented in (1) can be reproduced by quantum states that are unentangled. More specifically, Matsukevich and Kuzmich (1) make three explicit claims about entanglement: (i) entanglement between a single photon and a collective excitation in the two ensembles is created; (ii) a nearly maximally entangled state between the two ensembles is created; and (iii) a maximally entangled state between two photons would be created if one of the fields were not detected but instead were stored in an optical fiber.

It would be reasonable to expect to find these claims confirmed by appropriate measurements and by quantitative analysis of the resulting data that would substantiate quoting an experimentally determined degree of entanglement and associated uncertainty. Such an analysis, however, is absent in (1). Moreover, our analysis of this experiment arrives at rather different conclusions: First, we find that the data presented in (1) are insufficient to determine any value of the entanglement. Indeed, all results are consistent with unentangled states. Second, certain data have been discarded to produce figures 2 and 3 in (1), with the displayed “conditional probabilities” improperly defined by fiat. Third, crucial data necessary to quantify entanglement cannot be obtained by the experimental setup used in (1). Fourth, although Matsukevich and Kuzmich (1) suggest that their study has a strong connection to the results of (3) and draw upon that analysis, there are crucial differences between the two experiments. Most important, the criteria developed in (3), which Matsukevich and Kuzmich nevertheless employ, are not directly applicable to the results in (1).

In the remainder of this comment, we provide the main ideas behind our refutation of the three claims in (1) outlined above (4). We begin with a consideration of claim (ii)—the question of entanglement between two atomic ensembles—and then more briefly address claims (i) and (iii).

To address the issue of entanglement between the atomic ensembles, we begin by considering an entangled state of the form

$$\rho = \cos \theta \ket{0}\bra{1} + \exp(i\phi) \sin \theta \ket{1}\bra{0}$$

where $0,1$ refer to the number of excitations (either atomic or photonic) of two particular modes (atomic or electromagnetic). For $\theta = \pi/4$, this state is maximally entangled for any value of $\Psi$, which is the form of the state that Matsukevich and Kuzmich (1) claim to produce for the two atomic ensembles. It is important to note that the state in Eq. 1 is an idealization of the actual state one would and should ascribe to the system under investigation. In reality, the state will inevitably not be a pure state but a mixed state specified by a density operator $\rho$. Moreover, the actual system also will contain excitations in other, unwanted, modes. There will necessarily be finite probabilities for zero ($\ket{00}$) and multiple ($\ket{11}$) excitations in the atomic and field modes (2).

Matsukevich and Kuzmich (1) attempt to verify entanglement in three steps: first, by varying the angle $\theta$; second, by mapping the (presumed) state $\ket{\Psi}$ for atomic excitations onto the polarization state of a single photon; and, third, by performing a measurement on the state of the resulting photon while keeping only conclusive results. Unfortunately, each of these three steps contains a flaw. First, the natural and usual way to verify entanglement would be to vary the phase $\Psi$ for a fixed value of the angle $\theta$—indeed, this is the method employed in (3), with $\cos \theta = \sqrt{1/3}$. The strategy is to maintain (nearly) maximal entanglement while probing coherence between the two terms $\ket{0}\ket{1}$ and $\ket{1}\ket{0}$ by varying $\Psi$. In contrast, varying the angle $\theta$, as in (1), changes any entanglement from maximal ($\theta = \pm \pi/4$) to zero ($\theta = 0,\pi/2$).

Second, mapping the full state $\rho$ onto the two-dimensional polarization state of a single photon obviously precludes the determination of critical characteristics of the joint state of the atomic ensembles, which inhabits a Hilbert space of much higher dimensions. Moreover, Matsukevich and Kuzmich (1) provide no analysis that connects photon detection probabilities to the requisite probabilities that appear in the density matrix $\rho$ for the state of the atomic ensembles.

Third, by omitting data that correspond to detection events in which no photon was recorded, the authors can, in principle, make no definite estimate of the entanglement in the state $\rho$. To see this in a straightforward manner, consider that instead of the state $\ket{\Psi}$ presumed by Matsukevich and Kuzmich, an alternative state $\ket{\Psi'}$ is actually produced, where

$$\ket{\Psi'} = (\ket{0} + \epsilon \cos \theta \ket{1}) \times (\ket{0} + \epsilon \exp(i\phi) \sin \theta \ket{1})$$

with $\epsilon \ll 1$, corresponding to the conditions in (1). The state $\ket{\Psi'}$ is a product state and hence is unentangled. However, a measurement of $\ket{\Psi'}$ that keeps only those data arising from the two terms $\ket{0}\ket{1}$ and $\ket{1}\ket{0}$, while discarding null ($\ket{0}\ket{0}$) and higher order ($\ket{1}\ket{1}$) events, is indistinguishable from one that starts with the entangled state $\ket{\Psi}$ in Eq. 1 and ignores nondetection events, as is the case in (1). Thus, all data in (1) can in principle be obtained from unentangled states between the two ensembles (4).

Apart from these objections to the procedures followed in (1), it is useful to compare the methods and results of study with those of (3). For instance, figures 2 and 3 in (1) appear very similar to figures 3 and 4 in (3), but there are certain crucial differences. First is the fact, mentioned above, that a physically different angle is varied (instead of $\Psi$). Second, the measurements from the figures in (3) are converted explicitly to matrix elements of the density matrix, which in turn are converted into an expression for entanglement. By contrast, in (1), either no analysis is given (for the entanglement between atomic ensembles) or the method of analysis is not specified (for the entanglement between signal and idler photons). We emphasize here that the methods from (3) cannot be directly applied to (1) because a different phase is varied, in addition to the aforementioned mismatch in Hilbert-space dimension. Third, whereas the conditional probabilities plotted in figures 3 and 4 in (3) can indeed be of order unity because of the high efficiency for determining the internal state of a single trapped ion, in (1) they are smaller by three orders of magnitude (4). Nevertheless, the “conditional probabilities” plotted and quoted in (1) are given as

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being of order unity. Finally, the two sets of data in figure 3 in (3) are independent, whereas the similar-looking data of figure 2 in (1) are not independent—one set of data is just 1 minus the other.

We now return to claims (i) and (iii) in (1)—that is, the claims about entanglement between a photon and a joint collective excitation in the two ensembles, and entanglement between two photons. In both cases, there is never such an entangled state (4). The authors refer to a situation in which a state is produced that is a coherent superposition (in the ideal case) or a mixed state (in the generic case) of two states, one state where no excitations are present and one where two excitations are present with small probability \( p \). For \( p \ll 1 \), such a state has almost no entanglement [such small amounts of entanglement can still be detected by means of Clauser Horne Shimony Holt (CHSH) inequalities (4)]. However, if one could project out the “vacuum part,” then the remaining state with two excitations would be entangled in the ideal case (in the generic case, one still would have to check whether any entanglement would remain in the mixed state). In their experiment, Matsukevich and Kuzmich (1) sometimes detect one of the excitations and infer that there must have been an entangled state. Clearly, however, no such state was present before the measurement (because the vacuum part was still there) or after the measurement (because the detected excitation, a photon in fact, is always destroyed). The remark that storing instead of detecting the field would produce a (nearly) maximally entangled state is also incorrect, because in this case the vacuum part still would be present as well. Only if one could perform an extremely difficult quantum-nondemolition (QND) measurement (that is, a measurement of the presence of a photon without destroying it) would one approach a maximally entangled state.

We therefore conclude that, although Matsukevich and Kuzmich (1) provide important progress in the quest to develop the tools necessary for implementation of scalable quantum networks, the principal claims in the study are, unfortunately, largely unfounded. The measurements reported do not justify the claim of the production of “nearly maximally entangled states” between signal and idler photons, between two atomic ensembles, or between the signal photon and the two ensembles. The data reported are consistent with a wide class of unentangled states between the two ensembles.

Absent measurements to substantiate a convincing case for entanglement, the sweeping assertion in (1) that the reported capabilities “constitute a complete set of tools required to build an arbitrary large-scale quantum network” is without credibility.

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References and Notes
4. The reader is referred to the supporting online material for technical details.
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Supporting Online Material
www.sciencemag.org/cgi/content/full/309/5738/1187b/DC1
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References
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