Comment on “A Semi-Empirical Approach to Projecting Future Sea-Level Rise”

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Rahmstorf (Reports, 19 January 2007, p. 368) used the observed relation between rates of change of global surface temperature and sea level to predict future sea-level rise. We revisit the application of the statistical methods used and show that estimation of the regression coefficient is not robust. Methods commonly used within econometrics may be more appropriate for the problem of projected sea-level rise.

Rahmstorf (1) convincingly argued for the use of semi-empirical models for estimating sea-level response to future warming of the climate system. He hypothesized that the rate of global sea-level change is proportional to the global surface temperature departure from its equilibrium value. This hypothesis was statistically tested on observational data and a correlation coefficient of 0.88 was reported along with the global surface temperature departure from its equilibrium value. He hypothesized that the regression slope of both series was determined by linear extrapolation based on the nearest 15 years of data. The nonrobust trend of mean sea level was subsequently differentiated by calculating, at each point, the slope of a ±5-year least squares fit to obtain a “rate of sea level change” series. The correlation coefficient ρ between two series x_t and y_t, with deterministic time trends is defined as

\[ \rho = -\frac{E[(x_t - E(x_t))(y_t - E(y_t))]}{\sqrt{E[(x_t - E(x_t))^2]E[(y_t - E(y_t))^2]}} \]

where E denotes expectation value. The correlation coefficient thus measures the degree to which there are coincident departures of the two time series from their respective expectation values. When estimating the correlation coefficient between filtered versions of temperature and rate of sea-level change, Rahmstorf (1) substituted the expectation values by the sample average. This assumes stationarity of the series (4), which is obviously violated by the two series. As an illustration, when redoing the analysis, we approximated the expectation values of the two series by the more realistic choice of a linear trend. The estimated correlation coefficient then drops to 0.68. In addition, the corresponding regression coefficient increases from 3.3 mm/year per °C to 5.8 mm/year per °C. This nonrobust result underscores that the issue of correct statistical modeling is not an academic one and raises questions about the model put forward in (1).

Next, there is the point of establishing the significance of the correlation coefficient found, that is, “how likely is it to get the result by pure chance?” Rahmstorf appears to have estimated the P value of the correlation coefficient (1.6 × 10^{-4}) using a Student’s t distribution assuming 24 degrees of freedom. The number of degrees of freedom apparently comes from assuming that the 24 bins of 5-year length are statistically independent. However, both data series were low-pass filtered in (1) “by computing nonlinear trend lines, with an embedding period of 15 years.” Because of the autocorrelation introduced by the averaging procedure, neighboring 5-year bins can not be assumed to be statistically independent. A better approximation would be to set the number of degrees of freedom equal to the effective sample size calculated as 120/15 = 8 (4). Using our value of 0.68 for the correlation coefficient, we get a corresponding P value of 0.97.

Finally, Rahmstorf used a t test for making inferences about the correlation coefficient. This is based on the assumption of independent and identically distributed (i.i.d.) data, yet the data analyzed in (1) are, due to the trend, not i.i.d. This reservation also applies to the estimated confidence interval and, therefore, the range of projected sea-level changes by the year 2100. A thorough analysis of the problem would include the application of difference stationary time series analysis methods (5), for which there is a rich tradition in the field of econometrics. Such analysis may also be helpful to other problems in climate science.

References

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Editor's Summary

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