verse order, as in this case, is the real essence of the Heisenberg uncertainty principle. Besides its fundamental importance, the experimental implementation of such a sequence of basic quantum operations is an essential tool for the full-scale engineering of a quantum light state optimized for a multitude of different tasks (15), including robust quantum communication. As any quantum operation, including non-Gaussian operations, is composed of photon additions and subtractions (i.e. it can be expressed as $\langle \hat{a}, \hat{a}^\dagger \rangle$), our experimental results constitute a step toward the full quantum control of a field and the generation of highly entangled states (16).

References and Notes
2. Materials and methods are available as supporting material on Science Online.
14. Note that the positivity of the WF in the case of the add-then-subtract sequence is not due to experimental imperfections. As for squeezed states, the resulting state can be shown to be nonclassical (with a negative $P$ function), although possessing a positive WF.
17. We thank F. T. Arecchi, A. Montina, and E. Park for helpful comments and for a critical reading of the manuscript, and P. Poggi for the improvement of the results presented in this work. We wish to thank Enrico Persico for the technical support from the UK Engineering and Physical Sciences Research Council (EPSRC) and Quantum Information Processing Interdisciplinary Research Centre (QIPIRC).

Symmetrized Characterization of Noisy Quantum Processes
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A major goal of developing high-precision control of many-body quantum systems is to realize their potential as quantum computers. A substantial obstacle to this is the extreme fragility of quantum systems to "decoherence" from environmental noise and other control limitations. Although quantum computation is possible if the noise affecting the quantum system satisfies certain conditions, existing methods for noise characterization are intractable for present multibody systems. We introduce a technique based on symmetrization that enables direct experimental measurement of some key properties of the decoherence affecting a quantum system. Our method reduces the number of experiments required from exponential to polynomial in the number of subsystems. The technique is demonstrated for the optimization of control over nuclear spins in the solid state.

Quantum information enables efficient solutions to certain tasks that have no known efficient solution in the classical world, and it has reshaped our understanding of computational complexity. Harnessing the advantages of the quantum world requires the ability to robustly control quantum systems and, in particular, counteract the noise and decoherence affecting any physical realization of quantum information processors (QIPs). A pivotal step in this direction came with the discovery of quantum error correction codes (QECCs) (1, 2) and the threshold theorem for fault-tolerant (FT) quantum computation (3–6). To make use of quantum error correction and produce fault-tolerant protocols, we need to understand the nature of the noise affecting the system at hand. There is a direct way to fully characterize the noise using a procedure known as process tomography (7–9). However, this procedure requires resources that grow exponentially with the number of subsystems (usually two-level systems called "qubits") and is intractable for characterizing the multi-qubit quantum systems that are presently realized.

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Fig. 4. Experimental WFs (corrected for detection inefficiency) for (A) the original thermal state; (B) the photon-added-then-subtracted state; (C) the photon-subtracted-then-added state. (D) presents sections of the above Wigner functions (black squares correspond to the thermal field; red circles and blue triangles correspond to the photon-added-then-subtracted and the photon-subtracted-then-added states, respectively), together with the corresponding theoretical predictions (solid curves) (2).

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Materials and Methods
References and Notes
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(10–12). We introduce a general symmetrization method that allows for direct experimental characterization of some physically relevant features of the decoherence and apply it to develop an efficient experimental protocol for measuring multi-qubit correlations and memory effects in the noise. Compared with existing methods (13), the protocol yields an exponential savings in the number of experiments required to obtain such information. In the context of applications, this information enables optimization of error-correction strategy and tests of some assumptions underlying estimates of the FT threshold. Moreover, the estimated parameters are immediately relevant for optimizing experimental control methods.

Focusing on a system of n qubits, a complete description of a general noise model \( \Lambda \) requires \( O(2^n) \) parameters. Clearly an appropriate coarse-graining of this information is required; the challenge is to identify efficient methods for estimating the features of practical interest. The method we propose is based on identifying a reduced number of independent parameters. Distinct symmetry groups [represented by (a) red and (b) blue] uniformize different subsets of parameters.

![Fig. 1. Schematic of coarse-graining by symmetrization. Averaging the noise \( \Lambda \) by twirling under a symmetry group yields an effective noise process that has a reduced number of independent parameters. Distinct symmetrization groups [represented by (a) red and (b) blue] uniformize different subsets of parameters.](http://science.sciencemag.org/)  

Table 1. Summary of experimental results. The first four sets of experiments (three sets on the two-qubit liquid-state system and one on the three-qubit solid-state system) were designed to characterize the performance of the protocol under engineered noise. The final two sets demonstrate characterization of the (unknown) natural noise affecting the quantum memory created by multiple-pulse time-suspension sequences with different pulse spacings.

<table>
<thead>
<tr>
<th>No.</th>
<th>System</th>
<th>Map description</th>
<th>Kraus operators ( (A_i) )</th>
<th>( k )</th>
<th>( p_0 )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CHCl₃</td>
<td>Engineered: ( p = [0,1,0] ). ( \frac{1}{2} { Z_1, Z_2 } )</td>
<td>288</td>
<td>0.00000 ±0.0004</td>
<td>0.991 ±0.009</td>
<td>0.009 ±0.017</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>CHCl₃</td>
<td>Engineered: ( p = [0,0,1] ). ( { Z_1 Z_2 } )</td>
<td>288</td>
<td>0.00011 ±0.00006</td>
<td>0.0046 ±0.004</td>
<td>0.996 ±0.011</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>CHCl₃</td>
<td>Engineered: ( p = [0,1/2,1/2] ). ( { \exp[2i Z_1 Z_2] } )</td>
<td>288</td>
<td>0.254 ±0.010</td>
<td>0.995 ±0.019</td>
<td>0.004 ±0.004</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>C₂H₅O₄</td>
<td>Engineered: ( p = [0,1,0,0] ). ( \frac{1}{2 \sqrt{3}} { Z_1 Z_2 Z_3 } )</td>
<td>432</td>
<td>0.01 ±0.01</td>
<td>0.99 ±0.03</td>
<td>0.01 ±0.01</td>
<td>0.00 ±0.03</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>C₂H₅O₄</td>
<td>Natural Noise (i) unknown</td>
<td>432</td>
<td>0.44 ±0.02</td>
<td>0.45 ±0.03</td>
<td>0.10 ±0.08</td>
<td>0.01 ±0.03</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>C₂H₅O₄</td>
<td>Natural Noise (ii) unknown</td>
<td>432</td>
<td>0.84 ±0.01</td>
<td>0.15 ±0.03</td>
<td>0.01 ±0.01</td>
<td>0.00 ±0.02</td>
<td>-</td>
</tr>
</tbody>
</table>
where the matrix $\Omega_{cw}^{-1}$ is a matrix of combinatorial factors (SOM text). If in each single-shot experiment, the Clifford operators are chosen uniformly at random, then with $K = O [\log(2n)/\delta^2]$ experiments we can estimate each of the coefficients $c_w$ to precision $\delta$ with constant probability. All imperfections in the protocol contribute to the total probabilities of error. The protocol can be made robust against imperfections in the input state preparation, measurement, and twirling by factoring out the imperfections in the input state preparation, protocol contribute to the total probabilities of error. However, again the converse does not hold; consistency with this scaling does not guarantee that the untwirled noise obeys the Markovian semigroup property.

When applying $\{c_w\}$ to estimate $\{p_w\}$, the statistical uncertainty for $p_w$ grows exponentially with $w$ (SOM text). This still allows for characterization of other important features of the noise. Specifically, the probability $p_w$ is directly related to the entanglement fidelity of the channel, so this protocol provides an exponential savings over recently proposed methods for estimating this single figure of merit (16, 24, 25). [For another approach, see (17)]. Hence, by actually implementing any given code, we can bound the failure probability of that code with only $O[\log(2n)/\delta^2]$ experiments and without making any theoretical assumptions about the noise. Moreover, on physical grounds, we may expect the noise to become independent between qubits outside some fixed (but unknown) scale $b$, after which the $p_w$ decreases exponentially with $w$. The scale $b$ can be determined efficiently with $O(n^2)$ experiments.

Although a characterization of the twirled channel is useful given the relevance of twirled channels in some fault-tolerant protocols (22), the failure probability of the twirled channel gives an upper bound to the failure probability of the original untwirled channel whenever the performance of the code has some bound that is invariant under the symmetry associated with the twirl. This holds quite generally in the context of the symmetry considered above because the failure probability of a generic distance-$(2r+1)$ code is bounded above by the total probability of error terms with Pauli weight greater than $r$, and this weight remains invariant under conjugation by any $C_i \in C_i^\Omega_{cw}$.

Our protocol is efficient also in the context of an ensemble QIP (19). We prepare deviations from the identity state of the form $p_w = Z^w \otimes 1^{\otimes (n-w)}$, with $w \in \{1, \ldots, n\}$; hence, the (non-scalable) preparation of pseudo-pure states is avoided. As illustrated in Fig. 2, the ensemble protocol consists of conjugating the process $A \rightarrow A_{\text{tw}}$ with a randomly chosen pair $(C_i, \pi_i)$ in each run, where $\pi_i$ is a random permutation of the qubits. For input operator $p_w$, the output is $\sigma_{w}^{\text{out}} = A_{\text{tw}}(p_w)$. Averaging the output operators $\sigma_{w}^{\text{out}}$ over $i$ and $s$ returns the input operator scaled by $c_w$.

We performed an implementation of the above protocol on both a two-qubit (cholorof orm CHC1) liquid-state and a three-qubit (single-crystal Malonic acid C3H4O3) solid-state nuclear magnetic resonance QIP (26). The results of these experiments are summarized in Table 1. Statistical analysis for one liquid-state set is shown in fig. S1 and for the final two solid-state sets in Fig. 3. The final two sets of (solid-state) experiments were performed to characterize the unknown residual noise occurring under (i) one cycle of a C48 pulse sequence (27) with 10 $\mu$s pulse spacing, and (ii) two cycles of C48 with 5 $\mu$s pulse spacing. The C48 sequence is designed to suppress the dynamics due to the system's internal Hamiltonian and could be used, for example, for quantum memory. The evolution of the system under this pulse sequence can be evaluated theoretically by calculating the Magnus expansion (28) of the associated effective Hamiltonian, under which the residual effects appear as a sum of terms associated with the Zeeman and dipolar parts of the Hamiltonian, including cross terms. Roughly speaking, effective suppression of the $k^{\text{th}}$ term of the Hamiltonian takes places when $\gamma_k t_k << 1$, where $\gamma_k$ is the strength of the term and $t_k$ is the rate at which it is modulated by the pulse sequence. Generally, shorter delays lead to improved performance unless there is a competing process at the shorter time scale. Although two repetitions of the sequence with the pulse spacing of 5 $\mu$s has twice as many pulses as the single sequence with the 10 $\mu$s spacing, the probabilities of one-, two-, and three-body noise terms all decrease substantially (Table 1). However, the averaging under the 5 $\mu$s falls short of ideal performance as a result of incomplete (heteronuclear) decoupling of the qubits (three carbon nuclei) from the envi-

![Fig. 3. Results for $p_w$ from experiments 5 and 6 in Table 1. Shown are projections of the four-dimensional likelihood function onto various probability planes. The asymmetry seen in some of the confidence areas is a result of this projection. The results for one cycle with 10 $\mu$s pulse spacing (experiment 5) are in red, and the results for two cycles with 5 $\mu$s spacing (experiment 6) are in blue.](http://science.sciencemag.org/)

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The hyperfine interaction of an electron with the nuclei is considered as the primary obstacle to coherent control of the electron spin in semiconductor quantum dots. We show, however, that the nuclei in singly charged quantum dots act constructively by focusing the electron spin precession about a magnetic field into well-defined modes synchronized with a laser pulse protocol. In a dot with a synchronized electron, the light-stimulated fluctuations of the hyperfine nuclear field acting on the electron are suppressed. The information about electron spin precession is imprinted in the nuclei and thereby can be stored for tens of minutes in darkness. The frequency focusing drives an electron spin ensemble into dephasing-free subspaces with the potential to realize single frequency precession of the entire ensemble.

The possibility of encoding quantum information in the spins of quantum dot (QD) electrons has attracted considerable attention (1, 2). The spatial confinement protects the spins against the primary relaxation mechanisms in bulk, all of which arise from coupling of spin and orbital momenta. However, the electron hyperfine interaction with the lattice nuclei is enhanced by confinement, leading to spin decoherence and dephasing (3–10) and thus posing severe difficulties for processing quantum information. General schemes for suppressing decoherence have been discussed already (11). Electron spin relaxation in QDs may be overcome by polarizing the nuclear spins (12, 13), but the high degree of polarization required, close to 100% (12), has not been achieved yet (14–16).

We find that the hyperfine interaction, rather than being detrimental, can be used as a precision tool by demonstrating that it modifies the continuous mode spectrum of the electron spin precession in a QD ensemble into a few discrete modes. The information on this digital spectrum can be stored in the nuclear spin system for tens of minutes because of the long nuclear memory times (17, 18).

In a QD ensemble, fast electron spin dephasing arises not only from nuclear field fluctuations but also from variations of the electron g factor, leading to different spin precession frequencies. The dephasing due to these unavoidable variations can be partly overcome by mode-locking (19), which synchronizes the precession of specific electron spin modes in the ensemble with the clocking rate of a periodic pulsed laser. Still, it leaves a substantial fraction of dephased electron spins, whose precession frequencies do not satisfy the mode locking conditions. We demonstrate that the nuclear spin polarization adjusts the electron spin precession frequency in each quantum dot such that the whole ensemble becomes locked on very few frequencies.

The experiments were done on an ensemble of self-assembled (In,Ga)As/GaAs QDs (19, 20), each dot containing on average a single electron (21). The electron spin precession about a perpendicular magnetic field was studied by a pump-probe Faraday rotation (FR) technique with ps time resolution (22). Spin coherence is generated

References and Notes
26. Materials and methods are available as supporting material on Science Online.
29. This work benefited from discussions with R. Blume-Kohout, R. Cleve, M. Ditty, D. Gottesman, E. Knill, B. Levi, and A. Nayak and was supported by the Natural Science and Engineering Research Council of Canada (NSERC) grants 250673 and 327778, Ontario Research Development Challenge Fund (ORDCF) grant 323230-05, Army Research Office/Laboratory for Physical Sciences (ARO/LPS) grant W911NF-05-1-0469, and Army Research Office/Army of Information Technology and Complex Systems (AROMATICS) grant W911NF-05-1-0298.

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Nuclei-Induced Frequency Focusing of Electron Spin Coherence
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The experiments were done on an ensemble of self-assembled (In,Ga)As/GaAs QDs (19, 20), each dot containing on average a single electron (21). The electron spin precession about a perpendicular magnetic field was studied by a pump-probe Faraday rotation (FR) technique with ps time resolution (22). Spin coherence is generated
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