Comment on “Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture”

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Lu and Steinhardt (Reports, 23 February 2007, p. 1106) claimed the discovery of a large, potentially quasi-crystalline Islamic tiling in the Darb-i Imam shrine but regard the earlier Maragha tiling, previously described as quasiperiodic, as a small isolated motif. We demonstrate that the Darb-i Imam pattern is periodic and that the quasi-crystalline discs superimposed on its lattice are derivatives of the Maragha pattern.

When did quasi-crystalline patterns appear in Islamic architecture? Lu and Steinhardt (1, 2) reported the discovery of quasi-crystalline tiling patterns on the Darb-i Imam shrine, Isfahan, Iran (1453 C.E.), yet these patterns are not profoundly different from the previously described patterns of the Gunbad-i Kabad tomb tower in Maragha, Iran (1197 C.E.) (3). At both localities, Islamic artisans created decagonal/pentagonal quasiperiodic patterns starting from a central point and developed them as a cartwheel pattern with tenfold point group symmetry. In the patterns of the Darb-i Imam shrine (1, 4), such quasi-crystalline discs were placed in the (½,0) and (0,½) nodes of a periodic, centered net (continuous lines in Fig. 1) with plane group symmetry cmm [perpendicular reflections and half-turns; see also figures 3C and S7A in (2)]. The Maragha pattern is a quasiperiodic cartwheel pattern (3) spread over two adjacent panels of the building, with the origin at an upper corner of the building (Fig. 2A). A full 180-degree segment of a quasi-crystalline cartwheel pattern is always present (1). Its repetition around the eight-sided building is an architectural rather than a geometric problem. The half-cartwheel present at Maragha consists of over 400 tiles, whereas the Darb-i Imam half-cartwheel contains about 6% more tiles at its most optimistic delineation. Therefore, these cartwheel discs are large enough to demonstrate their quasi-crystalline character. Pattern adjustments along the contact of two adjacent cartwheel patterns are the only departures from quasiperiodicity in the Maragha patterns (Fig. 2A).

In the Darb-i Imam pattern, the space between four adjacent cartwheel discs with radial symmetry (each of them situated in the interior of a large-scale 10-fold star in Fig. 1) is occupied by 3 × 2 periods of a small-scale cmm pattern. Variability of infill of the small 10-fold stars present in these portions is discussed further below. A lozenge-

![Fig. 1. Portal of the Darb-i Imam Mosque (1453) in Esfahan. Translation identity of selected elements of the large-scale cmm net is indicated by coloring. Large 10-fold stars are Wyckoff positions (½,0) green, lozenges in (0,0) red, whereas that of selected small 10-fold stars is shown by correspondence of inscribed numbers. The tympanum contains one unit mesh of the cmm pattern, with the origin in the lower left-hand corner and two upper nodes truncated.](http://science.sciencemag.org/content/sci/318/5858/1383a/F1.large.jpg)
nearly identical both in tile arrangement and in size (the largest radius of these disc patterns reaches a prominent local 10-fold configuration). Both of them allow inflation/deflation inside a cartwheel and are representative fragments of quasi-crystalline decagonal cartwheel patterns, either adjusted to the architecture of the building at their periphery or embedded in a periodic framework. The Darb-i Imam cartwheel discs appear to be later variations of the tiling scheme from Maragha. This corroborates the suggestion by Makovicky (3), who dated the discovery of quasi-crystalline patterns to the years 1196 and 1197—much earlier than Lu and Steinhardt (1)—and confirms the exceptional cultural value of the Gunbad-i Kabud tower in Maragha.

In broad analogy to the present-day situation in Morocco, we believe that the artisans were satisfied by creating a large fundamental domain without being concerned with a mathematical notion of infinitely expandable quasiperiodic patterns. However, they understood and used to their advantage some of the local geometric properties of the quasi-crystalline patterns they constructed.

References and Notes
5. G. Schneider, Geometrische Bauornamente der Seldschuken in Kleinasien (L. Reichert Verlag, Wiesbaden, 1980).
6. The comments of three anonymous reviewers are gratefully acknowledged.

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