Comment on “Multiparteite Entanglement Among Single Spins in Diamond”

Brendon W. Lovett* and Simon C. Benjamin*

Neumann et al. (Reports, 6 June 2008, p. 1326) recently reported the preparation of multiparticle entanglement among single spins in diamond. However, two of the system’s nuclear eigenstates are incorrectly described as product states when they are inherently entangled. Consequently, three of the six states reported, namely the odd-parity Bell states and the W state, were not actually produced.

Neumann et al. (1) reported the creation of quantum entanglement among the spins in the vicinity of a single nitrogen-vacancy (NV) defect in diamond. The work demonstrates the complex interplay between different degrees of freedom in this solid state system. However, we believe that certain important eigenstates of the NV system have been wrongly classified as products of single-spin states. We argue that these eigenstates are inherently entangled and, therefore, although the authors have demonstrated manipulation of these states, they did not introduce entanglement.

Neumann et al. made an unambiguous claim as to the two-spin states they generated. They stated that “All four maximally entangled states, namely the Bell states \( \Phi^- = 1/\sqrt{2}(|00\rangle \pm |11\rangle) \) and \( \Psi^- = 1/\sqrt{2}(|01\rangle \pm |10\rangle) \) are generated, where ‘0’ and ‘1’ denote the two possible nuclear spin orientations \( m_I = -1/2 \leftrightarrow |0\rangle, m_I = +1/2 \leftrightarrow |1\rangle, |N_1N_2\rangle\).”

The spin product states \{00\}, \{01\}, \{10\}, \{11\}\) thus defined are indeed the appropriate choice for a study aimed at entanglement generation; entanglement is a physical resource that exists between distinct entities (2), and the relevant entities here are the nuclear (and electron) spins. The quoted definition directs the reader to figure 1B in (1), where these product states are assigned to four eigenstates of the system. As we show here, however, two of these assignments are incorrect; consequently, in creating a superposition of the system’s eigenstates, the authors do not actually create the \( \Psi^- \) states. In actuality, those very Bell states are system eigenstates before any manipulations, and the applied manipulations evidently disentangle the spins. The authors also discuss tripartite-entangled quantum states, in which the third party is the electron spin associated with the NV defect. They assert that a GHZ state and a \( W \) state are generated. However, the latter claim, because it involves the same odd-parity nuclear spin states, is also incorrect.

We first classify the eigenstates of the electron-nuclear spin system. The supporting online material for (1) presents the following Hamiltonian for the single electron spin \( (S = 1) \) coupled equally to two surrounding \(^{13}\text{C}\) spins \( (I = 1/2) \):

\[
H = g_e \beta_B \vec{B} \cdot \vec{S} + \sum_i (\mathbf{\hat{S}} \cdot \mathbf{A} \cdot \mathbf{I}_i - g_e \beta_B \mathbf{I}_i^2)
\]

(1)

where \( \mathbf{A} = A_z \mathbf{I} \) (our definition) and \( g_e \beta_B \mathbf{I}^2 \) is the electron spin and the second term is the zero field splitting, which lifts the degeneracy of the \( S_z \) ensemble of these two identical spins; the third term is the hyperfine coupling between each nuclear spin and the electron, which is described by a tensor \( \mathbf{A} \). The tensor is assumed uniaxial with principal components lying along the three Cartesian axes \( x, y, z \) and \( A_1 = A_\alpha = A_{xy}, A_2 = A_z = A_\beta \). The fourth term is the Zeeman splitting for the nuclear spins.

The Hamiltonian commutes with the operator \( \mathbf{J}_z = S_z + I_1 + I_2 \) (our definition); hence this quantity is conserved. Therefore the Hamiltonian splits into separate uncoupled subspace characterized by distinct values of \( J_z \). Referring to figure 1B in (1), we would like to discuss the middle two states of the four states within the \( m_s = -1 \) manifold (\( m_s \) in that figure corresponds to the \( S_z \) used in the supplementary information). These two states have \( J_z = -1 + 1/2 - 1/2 = -1 \).

The Hamiltonian for the \( J_z = -1 \) subspace spans a three dimensional Hilbert space. Using the notation \( |S_\alpha I_1 I_2\rangle \), a suitable basis is \( \{|-1, 1/2, -1/2\}, \{-1, -1/2, 1/2\}, \{0, -1/2, -1/2\}\). In a matrix form in this basis the Hamiltonian becomes

\[
H_{J_z = -1} = \begin{pmatrix}
E_0 & 0 & K \\
0 & E_0 & K \\
K & K & E_1
\end{pmatrix}
\]

(2)

The quoted definition directs the reader to figure 1C. In the reduced notation for the nuclear spins used in (1), defined for the \( m_s = -1 \) subspace, these eigenstates become

\[
|\psi_+\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle),
\]

\[
|\psi_-\rangle = \frac{1}{\sqrt{2}} (|10\rangle - |01\rangle)
\]

(5)

Importantly, these are not the separable states shown in the scheme of figure 1B in (1); rather, they are entangled Bell states. We thus conclude that the eigenstates of the system in this subspace are already maximally entangled. The result of making an equal superposition of these two states \( \frac{1}{\sqrt{2}} (|\psi_+\rangle \pm |\psi_-\rangle) \), for which Neumann et al. present evidence in figure 2, B and C (1), is nothing but a construction of the separable states \( |10\rangle \) and \( |01\rangle \). The definition of the nuclear spin eigenstates also impacts on the definition of the \( W \) state discussed as the conclusion of (1) and in figure 4.

References


22 August 2008; accepted 2 February 2009

1169c

www.sciencemag.org  SCIENCE VOL 323 27 FEBRUARY 2009
Comment on "Multipartite Entanglement Among Single Spins in Diamond"
Brendon W. Lovett and Simon C. Benjamin (February 27, 2009)
Science 323 (5918), 1169. [doi: 10.1126/science.1168458]

Editor's Summary

This copy is for your personal, non-commercial use only.

Article Tools
Visit the online version of this article to access the personalization and article tools:
http://science.sciencemag.org/content/323/5918/1169.3

Permissions
Obtain information about reproducing this article:
http://www.sciencemag.org/about/permissions.dtl