THE INFLUENCE OF FOURIER'S SERIES UPON THE DEVELOPMENT OF MATHEMATICS

In selecting a subject for to-day's address I have had the difficult task of interesting two distinct classes of men, the astronomer and the mathematician. I have therefore chosen a topic which, I trust, will appeal to both—trigonometric series. Though I propose to treat it only in its mathematical aspects, I shall try to do so in a broad way, tracing its general influence upon the trend of mathematical thought.

As you know, the theory of the infinite trigonometric series,

\[ f(x) = \sum_{n=0}^{\infty} \left[ a_n \cos nx + b_n \sin nx \right] \]

is different \textit{ab initio} from that of the power series,

\[ P(x) = c_0 + c_1(x - a) + c_2(x - a)^2 + \ldots. \]

For the latter the fundamental element is \( x^n \), of which the graph is, for positive \( x \), a monotone increasing function, wholly regular, without peculiarities of any sort. It is therefore in no way surprising that the power series obtained by combining terms of form \( c_n x^n \) define the most civilized members of mathematical society—the so-called analytic functions—which are most orderly in their behavior, being continuous throughout their "domains," possessing derivatives of all orders and a Taylor's series at every point; and so forth. On the other hand, the graph of \( \sin nx \) or \( \cos nx \) is a wave curve with crests and troughs, whose number in any \( x \) interval increases indefi-