THE ALGEBRAIC NUMBERS AND DIVISION¹

As to what should most appropriately be the character of an address delivered on an occasion such as the present I am not quite certain. Whether it were better that one should be somewhat general and discursive in his remarks or whether he would be justified in offering a considerable amount of highly specialized and technical material I hardly know. Possibly a critical or historical survey of some subject would be more in place, or again under circumstances might perhaps be permitted to the speaker to discuss some phase or aspect of a special field which would afford opportunity to present, among others, results obtained by himself or to develop methods employed in his investigations.

On this occasion I shall venture to say a little about a subject in which I have had a special interest, but in which results obtained are of some years' standing, since with the routine of teaching, executive activities in various connections and a vast amount of organizing work have, in recent years, combined to prevent productive effort and have interfered with the formulation for publication of results already there.

What I have to say will consist largely in statements. There will be no attempt to give proofs. These will be available elsewhere. A certain amount of recapitulation of more or less familiar facts will be necessary in order to establish connectivity, and as a preliminary to the later statements. The net result, I trust, will be found to contain an element of novelty.

An integer can be represented as a product of powers of primes, the exponents being positive integers. A rational fraction can be represented as a product of powers of primes, the exponents being positive or negative integers. The exponent of a given prime in the representation of a rational number as a product of powers of primes we call the order number of the rational number for the prime in question. We say that 0 is the order number of a rational number for a prime which does not appear explicitly in the representation of the number as a product of powers of primes. One rational number we say is divisible by another when the quotient of the first number by the second is integral, otherwise

¹ Address of the retiring vice-president and chairman of Section A—Mathematics—American Association for the Advancement of Science, Kansas City, Mo., December, 1925.