A MATHEMATICAL CRITIQUE OF SOME PHYSICAL THEORIES

The purpose of this paper was to review some of the mathematical-physical theories of the past and of the present, indicating briefly the nature of certain concepts upon which these theories rest as well as attendant logical difficulties, and proposing certain modifications. It goes without saying that geometry is the first and simplest of such theories. Some day, when the field of knowledge has extended so far that simplification becomes necessary, ordinary geometry may be approached somewhat as follows:

1) Geometry treats of elements called points and the relation called distance between pairs of points.
2) The complete treatment of distances between pairs of points may be arranged as follows:

(a) The points P correspond to real number triples \((x, y, z)\);
(b) The squared distance between \(P_1\) and \(P_2\) is \((x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\).

All geometry follows very readily from these agreements. Beginning in this way one may successively define line-segments, lines, planes, perpendicularity, rectangular coordinate systems, etc. The whole body of geometrical fact with corresponding analytic framework is easily deducible, and yet one may stop at the fundamental principles without taking up beautiful but less vital geometrical studies. In its origin the geometrical concept of space is always to be associated with that of a corresponding body of reference.

Classical dynamics arises in the attempt to use Euclidean space and absolute time as the means for expressing the laws of nature. There lie certain fundamental difficulties at the very basis of this attempt to make space the container of matter. The simplest illustration of them arises in dealing with a collection of "equal rigid elastic spheres." When only two spheres collide, the assumed laws of contact action determine uniquely their directions and velocities after collision; but when more than two spheres collide, the situation is entirely different.

1 Synopsis of address as retiring president of the American Mathematical Society before a joint meeting of the American Association for the Advancement of Science, the American Mathematical Society and the Mathematical Association of America. The full text will appear in an early number of the Bulletin of the American Mathematical Society.