THE NOTION OF PROBABLE ERROR
IN ELEMENTARY STATISTICS:

What I have to say to-day is not addressed to professional mathematicians or statisticians. To mathematicians and statisticians all that I shall say is already entirely familiar. There are two other classes of readers, however, to whom I hope the discussion may be of service: (1) the rapidly increasing number of laymen who, without technical mathematical training, are constantly coming upon such terms as ‘probable error’ in their general reading, and (2) the non-mathematical research worker who is constantly tempted to embellish his numerical results by adding an array of “probable errors”—obtained, alas, too often by the simple process of substituting blindly in a formula. (A formula, of course, is an essential tool; what will concern us here, however, is the underlying significance of such a formula, and the necessary limitations surrounding the proper use of it.)

What are the principles that lie behind the common use of the term “probable error”? What does it really mean when we say, for example, that a quantity $x$ has an estimated value of 3.6 with a “probable error” of 0.2 (written $x = 3.6 \pm 0.2$)?

The conventional reply to this question will occur to all of us—namely, that “the probable error is the error that is as likely as not to be exceeded.” For example, if $x = 3.6 \pm 0.2$ the conventional understanding is that the “true value” of $x$ is as likely to lie outside the limits 3.4 and 3.8 as it is to lie between those limits.

But this conventional reply does not go very far behind the scenes—we should like to have something more fundamental. Under what circumstances can we properly speak of errors as “equally likely” to occur? What are the fundamental considerations underlying the whole range of ideas which are suggested by the term “probable error”? I believe the best modern opinion is in favor of treating the so-called “probable error” from the point of view of empirical statistics, with as little reference as possible to the technical theory of probability; and I am convinced that much misunderstanding will be avoided if we can keep as