A MATHEMATICAL THEORY OF EQUILIBRIUM WITH APPLICATIONS TO MINIMAL SURFACE THEORY

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The theory of equilibrium points or critical points of functions appears in fragmentary form in the work of Poincaré, Maxwell and Kronecker. Birkhoff introduced the minimax principle and applied it in dynamics. A systematic study of the critical points of functions of $n$-variables was begun by the author in 1922. A. B. Brown made important contributions. In 1924 the calculus of variations in the large was introduced and developed as an extension of the theory of critical points of $n$-variables. The integrals used were ordinary and were regarded as functions of the curves along which they were evaluated.

In 1937 the theory was put on a more general basis with the function defined on an abstract metric space. For details the reader may refer to the author's fascicule on "Functional Topology and Abstract Variational Theory," published by Gauthier-Villars, Paris.

In this general theory one is free from any limitation of dimension. It is immaterial whether the independent variable is a point in a Euclidean space, a curve, a surface or a more general configuration. A critical point of a function of $n$-variables is a point at which all the partial derivatives vanish. When the function is an integral in the calculus of variations—for example, the integral of length on a surface or the area of a surface bounded by a curve—a critical point (curve or surface) is one satisfying the Euler-La-
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