Supporting Online Material for

Crustal Dilatation Observed by GRACE After the 2004 Sumatra-Andaman Earthquake
Shin-Chan Han,* C. K. Shum, Michael Bevis, Chen Ji, Chung-Yen Kuo

*To whom correspondence should be addressed. E-mail: han.104@osu.edu

DOI: 10.1126/science.1128661

This PDF file includes:

Materials and Methods
Figs. S1 to S4
Table S1
References
1. Materials and Methods

1.1 Gravity change caused by uplift/subsidence

The sub-surface topography changes (vertical displacement) at the interfaces of different density materials, i.e., ocean/crust and crust/mantle, cause gravity changes (Fig. S1). In order to compute the displacement, a half-space below the seafloor is assumed to be elastic with a Poisson’s ratio of 0.25 (i.e., the same Lamé constants). \( h \) and \( d \) are the sea level height from the sea floor and crust thickness, respectively. \( \rho_s \), \( \rho_c \) and \( \rho_m \) are density of sea water, crust, and mantle, respectively. \( \mathbf{u}(\mathbf{r}) \) is a displacement vector at \( \mathbf{r} \), where \( \mathbf{r} = (x, y, z) \). \( t(x,y) \) and \( t'(x,y) \) indicate the amount of coseismic uplift/subsidence at the horizontal location \((x,y)\), at the seafloor and the Moho, respectively. We use 3 km and 20 km for \( h \) and \( d \), respectively, and 1025 kg/m\(^3\), 2750 kg/m\(^3\), and 3300 kg/m\(^3\) for \( \rho_s \), \( \rho_c \) and \( \rho_m \), respectively. The crust model indicates that \( d \) varies within the range of 10 km to 30 km \((J)\). \( \nabla \cdot \mathbf{u} \) is the divergence of a displacement vector which is essentially same as a volumetric dilatation (sum of the normal strains).

Based on the seismic fault model \((2)\) including magnitude of slips and rakes on each fault plane, we evaluated the vertical deformation at the surface of the half-space (i.e., the sea floor) and the coseismic change of the Moho topography \( t'(x,y) \) following \((3, 4)\). We then computed the gravity at the sea level which is approximately 3 km above the surface of the half-space. Since the height, \( h \), is much larger than the amount of uplift/subsidence \((\sim m)\), the gravity change at sea level can be approximated by using a thin layer with a surface density \( \sigma \), which is a volumetric density multiplied by the uplift/subsidence. The gravity change at sea level can be computed by upward continuation of the surface density change at seafloor and at Moho as follows \((5, 6)\):
\[ \Delta g_{cs}(x, y, h) = \mathcal{F}^{-1}\left\{ \mathcal{F}\{2\pi G\Delta \sigma_{cs}(x, y)\} \exp\left(-2\pi \sqrt{(f_x)^2 + (f_y)^2} h\right) \right\}, \] (1)

\[ \Delta g_{mc}(x, y, h + d) = \mathcal{F}^{-1}\left\{ \mathcal{F}\{2\pi G\Delta \sigma_{mc}(x, y)\} \exp\left(-2\pi \sqrt{(f_x)^2 + (f_y)^2} (h + d)\right) \right\}, \] (2)

where \( \mathcal{F}\{ \} \) and \( \mathcal{F}^{-1}\{ \} \) indicate the Fourier transform and its inverse transformation operator, respectively. \( f_x \) and \( f_y \) indicate the frequency in \( x \) and in \( y \), respectively. The surface density can be computed as follows:

\[ \Delta \sigma_{cs}(x, y) = (\rho_c - \rho_s)\delta(x, y), \] (3)

\[ \Delta \sigma_{mc}(x, y) = (\rho_m - \rho_c)\delta'(x, y). \] (4)

Since the Fourier operators are scale invariant, we factored the density parameter and constants out of the Fourier operators as follows:

\[ \Delta g_{cs}(x, y, h) = 2\pi G\mathcal{F}^{-1}\left\{ \mathcal{F}\{\delta(x, y)\} \exp\left(-2\pi \sqrt{(f_x)^2 + (f_y)^2} h\right) \right\}(\rho_c - \rho_s), \] (5)

\[ \Delta g_{mc}(x, y, h + d) = 2\pi G\mathcal{F}^{-1}\left\{ \mathcal{F}\{\delta'(x, y)\} \exp\left(-2\pi \sqrt{(f_x)^2 + (f_y)^2} (h + d)\right) \right\}(\rho_m - \rho_c). \] (6)

1.2 Gravity change caused by density change (dilatation)

The gravitational potential anomaly at \( \mathbf{r}' \), \( T(\mathbf{r}') \), is computed by integrating mass anomaly in the media as follows:

\[ T(\mathbf{r}') = G \int \frac{dm(\mathbf{r})}{|\mathbf{r}' - \mathbf{r}|}, \] (7)

where \( dm(\mathbf{r}) = \Delta \rho(\mathbf{r})dV \). Now we consider the mass conservation to compute the density anomaly in term of volumetric dilatation as follows:

\[ M = \rho V = (\rho + \Delta \rho)(V + dV), \] (8)

where \( M, \rho, \) and \( V \) indicate mass, density, and volume, respectively. It yields \( \rho dV + \Delta \rho V = 0 \) in the first order approximation. Therefore, we obtain a volumetric dilatation (sum of normal strain) as follows:

\[ \frac{dV}{V} = -\frac{\Delta \rho}{\rho} = \nabla \cdot \mathbf{u}, \] (9)
where \( \nabla \cdot \mathbf{u} \) is divergence of displacement vector \( \mathbf{u} \). Eq.(9) gives \( \Delta \rho = -\rho \nabla \cdot \mathbf{u} \) and consequently, \( dm(r) = -\rho(r) \nabla \cdot \mathbf{u}(r) dV \).

The gravitation potential and its vertical derivative (gravity) are written in terms of the divergence of displacement vector (7, 8) as follows:

\[
T(r') = -G \int \frac{\rho(r) \nabla \cdot \mathbf{u}(r)}{|r' - r|} dV,
\]

\[
\Delta g(r') = \frac{\partial T(r')}{\partial z'} = -G \frac{\partial}{\partial z'} \int \frac{\rho(r) \nabla \cdot \mathbf{u}(r)}{|r' - r|} dV.
\]

We may simplify the integral by specifying the density structure. That is, we divide the integral into two parts, i.e., contributions from the crust and mantle, by considering only the two distinct densities in the crust and mantle as follows:

\[
\Delta g(r') = G \rho_e \int \left[ 0^0 - \frac{\partial}{\partial z'} \frac{\nabla \cdot \mathbf{u}(r)}{|r' - r|} dS + G \rho_m \int \left( 0^d - \frac{\partial}{\partial z'} \frac{\nabla \cdot \mathbf{u}(r)}{|r' - r|} dS \right) .
\]

The both integrals are numerically evaluated by approximating them with sum of parallelepipeds with finite dimension. We evaluate dilatations at 8 corners of each parallelepiped and take an average to represent a mean dilatation within each parallelepiped.

1.3 Local Smoothing

In order to derive local Gaussian smoothing factor \( S_{i,j} \), we defined the following variables:

\[
w_G(\psi_{i,j}) = \exp(-a(1 - \cos \psi_{i,j})) ,
\]

\[
\cos(\psi_{i,j}) = \cos \theta_i \cos(\theta_i + s \Delta \theta) + \sin \theta_i \sin(\theta_i + s \Delta \theta) \cos(\lambda_i - \lambda_j) ,
\]

where \( a = 1/(1 - \cos \psi_0) \) and consequently \( w_G(\psi_0) = \exp(-1) \approx 0.37 \). \( \psi_0 \), scaled with the earth’s mean radius R, is called an averaging radius. \( \theta_i \) and \( \lambda_i \) are co-latitude and longitude of \( i \)-th location, respectively, and \( \Delta \theta = \theta_j - \theta_i \). Finally, \( s \) is the flattening factor which controls the ellipticity of the smoothing function. For example, if \( s \) is greater than 1, \( w_G \) has greater averaging radius along longitude than along latitude. If \( s \) is equal to 1, it results the isotropic Gaussian smoothing function.

Finally we define the local Gaussian smoothing function used in this study as follows:
\[ S_{i,j} = \begin{cases} 
 w_c(\psi_{i,j}) , & 0 \leq \psi_{i,j} \leq \psi_c \\
 0 , & \psi_{i,j} > \psi_c 
\end{cases} , \quad (15) \]

where \( \psi_c = 3\psi_0 / 2 \).
2. Supporting Figures

Fig S1. A simplified earth’s density structure used in this study for computing the gravity effects caused by the vertical displacement at the sea floor and the Moho and by density change in the crust and the mantle.
Fig S2. (A) Gravity changes (μGal) from GRACE monthly mean global spherical harmonic coefficients; 4-month (Feb. to May) mean difference between 2004 and 2003, (B) 4-month mean difference between 2005 and 2003, (C) 4-month mean difference between 2005 and 2004, (D) Gravity changes caused by the vertical displacement, predicted from the seismic model, same as Fig. 3C, but with a different averaging radius. (E) Those due to the dilatation, same as Fig. 3F, but with a different averaging radius. (F) Total gravity change predicted from the model, same as Fig. 4, but with a different averaging radius. The isotropic smoothing radius of 800 km was used.
Fig S3. The predicted gravity changes (µGal) from the seismic model (no smoothing applied, i.e., full resolution of 50 km). (A) uplift/subsidence at the sea floor, (B) uplift/subsidence at the Moho, (C) total effect of vertical displacement, i.e., (A) + (B), (D) expansion within the crust, (E) compression within the mantle, (F) total effect of dilatation, i.e., (D) + (E), and (G) total effect of vertical motion and dilatation, i.e., (C) + (F).
Fig S4. Slip distribution on 7 fault planes used in this study (2). Strikes of the Sumatra, Nicobar, and Andaman fault planes are 315°, 341°, 5°, respectively. Dip angles of the upper part of those fault planes are 7°, 8°, 9°, respectively. Dip angles of the lower part of those fault planes are 15°, 18°, 22°, respectively. Strikes and dip angles for the Nias fault plane are 325° and 15°, respectively.
3. Supporting Table

**Table S1.** Ratio of total power spectra

<table>
<thead>
<tr>
<th></th>
<th>C/A</th>
<th>D/B</th>
<th>A/B</th>
<th>C/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/A</td>
<td>0.02</td>
<td>0.49</td>
<td>25.95</td>
<td>1.12</td>
</tr>
</tbody>
</table>

A, B, C, and D indicate the total power of vertical displacement from the seismic model (Fig. S3C) and dilatation from the seismic model (Fig. S3F), smoothed vertical displacement (Fig. 3C), and smoothed dilatation (Fig. 3F), respectively.
4. References

2. C. Ji, A coseismic displacement model for the Sumatra-Andaman (24 December 2004) earthquake and a model for the Nias (28 March 2005) earthquake. They are derived from a combination of seismic and geodetic observations, and are formulated in terms of over 1000 slip (i.e. Burgers) vectors on 7 rectangular dislocation patches using the uniform elastic half-space formalism.