Supporting Online Material for

Long-Range Order in Electronic Transport Through Disordered Metal Films

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Magnetic field microscope

In the magnetic field microscope ($S1, S2$) atoms probe the local magnetic potential through their magnetic interaction. Bose Einsten Condensates (BEC) fill the magnetic potential landscape like a liquid, the chemical potential defining the 'surface' height, while ultracold non condensed 'thermal' atoms fill the potential landscape like a gas (see Figure S1). In both cases the local column density is a measurement of the local potential energy surface. If the atoms are trapped in a quasi one dimensional (1d) geometry, as employed here, the variation in the local 1d linear density $n_{1d}(x)$ is a direct measure of the variations in the trapping potential $\Delta V(x)$. For a BEC one finds

$$\Delta V(x) = -\hbar \omega_\perp \sqrt{1 + 4a_{\text{scat}} n_{1d}(x)}, \quad (S1)$$

where $2\pi \omega_\perp$ is the trapping frequency characterizing the transverse confinement and $a_{\text{scat}}$ is the atom-atom scattering length. For thermal non condensed atoms one finds

$$\Delta V(x) = -k_B T \log \frac{n_{1d}(x)}{\langle n_{1d} \rangle}, \quad (S2)$$

where $k_B$ is the Boltzmann constant. The normalization to the mean density makes the measured potential profiles $\Delta V(x)$ independent of the atom number, and the connection between the different profiles in the scan more direct.

The longitudinal potential variations are related to the magnetic field by $V(z) = m_F g_F \mu_B B(z)$ where $\mu_B$ is Bohr’s magneton, $m_F$ is the quantum number associated with the Zeeman state of the atom and $g_F$ is the Lande-factor.

For a BEC the measurement range is given by the chemical potential $\mu$. For a thermal gas the relevant range is a few times $k_B T$. In both cases the sensitivity of the measurement is a small fraction of this range. A chemical potential of $\mu = 1$kHz corresponds to 70 nT for an atom with a magnetic moment of $1 \mu_B$; Similarly a temperature of $1 \mu K$ corresponds to 1.3 $\mu$T. A 1d BEC gives a better sensitivity to very small variations. A sensitivity of 200 pT at 3 $\mu$m spatial resolution was obtained in ($S1, S2$). The sensitivity can be tuned by changing $\omega_\perp$. A thermal gas is less sensitive, but one can measure over a larger range and the measurement range can be
tuned easily by adjusting the temperature. Very good sensitivity can be reached especially when one works with very cold thermal atoms. A BEC may be more sensitive but the experimental methods and the required stability (constant chemical potential) are more demanding and the analysis becomes more cumbersome with the always present phase fluctuations in 1d quasi condensates.

Consequently, even though a BEC was used in the exploratory research at the beginning of this investigation, for the final analysis described in this report a BEC was not used. The ultracold thermal atoms were found to be a better probe because they gave a better homogeneity of the measurement, and they were able to cover the higher potential barriers. Also, as noted, controlling the temperature allowed to better tune the sensitivity over the required measurement range.

Imaging the density of the trapped 1d cloud allows to map out the spatial variation of the magnetic potential with high resolution over the entire length of the cloud simultaneously. Scanning the 1d atom cloud in the transverse direction allows to reconstruct a full two dimensional (2d) potential landscape with unprecedented sensitivity. For this two dimensional reconstruction the mean value of the magnetic potential has been subtracted for each scanned line. This procedure has been chosen, as the absolute value of the potential cannot be determined by this method.

Compared to scanning probes having a $\mu m$ scale spatial resolution and $10^{-5} T$ sensitivity, or superconducting quantum interference devices (SQUIDs) having $10^{-13} T$ sensitivity but a resolution of tens of $\mu m$, ultracold atom magnetometry has both high sensitivity ($10^{-10} T$) and high spatial resolution (several $\mu m$) (S2). In addition, ultracold atoms enable high resolution over a large length scale ($mm$) in a single shot. This enables the simultaneous observation of microscopic and macroscopic phenomena, as described in this work.

On atom chips (S3, S4) and close to a current carrying wire the minimum of the trapping potential lies exactly along the line where the transverse magnetic field $B_y$ generated by the homogeneous part of the current density $J_x^{(0)}$ is canceled by the homogeneous bias field, thus
forming the trap. The potential variations $\delta B_x(x)$ of the remaining homogeneous field $B_x$ along the wire axis are then directly related to a change in the local magnetic field direction ($S1, S2$). The signal is independent of the offset field $B_x$. Consequently magnetic field microscopy using ultracold atoms allows to detect extremely small variations of an arbitrary non-zero homogeneous offset field. This is a major advantage compared to other techniques.

We observe very small magnetic field variations $\delta B_x(x)$ (typically 1 nT) in a direction orthogonal to a large field $B_y$ (typically 1-10 mT), the latter canceled in forming the trap. Consequently we can observe with high spatial resolution very small changes in the magnetic field direction down to $\mu$rad sensitivity. This is another significant advantage of magnetic field microscopy using ultracold atoms compared to other techniques.

In our setup these variations in the magnetic field direction are directly linked to changes in the local current flow pattern that creates these fields. The variations in the current flow direction manifest themselves in small spatially varying transverse components $J_y(x)$ of the current $J^{(0)}$.

From a measured 2d map of the magnetic field one can reconstruct the local transverse current in the conductor, assuming a two dimensional current flow ($S2$). This reconstruction involves the deconvolution of Biot Savart’s law which can be performed if the current density is assumed to be confined to a 2d-plane. In this case only $J_y$ contributes to the magnetic field $B_x$ and can be calculated according to

$$J_y(x, y) = \frac{2}{\mu_0 d} \mathcal{F}^{-1} \{ \bar{B}_x(k_x, k_y) e^{i|z|} \}(x, y), \quad (S3)$$

where $\bar{B}_x(k_x, k_y) = \mathcal{F} \{ B_x(x, y) \}(k_x, k_y)$ and $\mathcal{F}$ indicates a two-dimensional Fourier transform. This assumes a current homogeneous within the thickness of the conducting film $d$

**The atom chip experiment**

Atom chips ($S5$) are microscopic devices where the electric and magnetic fields for atom manipulations are created by microscopic wires and electrodes nano fabricated on a substrate ($S3, S4$). The precision and robust alignment of the fabricated structures allow well controlled quantum
manipulations of ultracold atoms (S6). Our experiments are performed with ultra cold samples of $^{87}$Rb atoms magnetically trapped in the $|F = 2, m_F = 2\rangle$ state on such atom chips.

We prepare our ultracold atomic samples used to probe the magnetic field landscape on an atom chip using our standard procedure (S7, S8). $^{87}$Rb atoms are first laser cooled, then optically pumped into the $|F = 2, m_F = 2\rangle$ state, and finally cooled to a $1\mu K$ temperature in 20s of forced evaporation in a magnetic trap of transverse and longitudinal trapping frequencies $\omega_\perp = 2\pi \times 840\text{Hz}$ and $\omega_x = 2\pi \times 21\text{Hz}$. From this trap, situated at 100$\mu m$ above the chip surface, the atoms are transferred within a typical time of 800ms to the measurement location and further cooled to the final desired temperature selected to optimally measure the magnetic field variations. The resulting atom cloud is then typically 800 $\mu m$ long, trapped in a quasi one dimensional geometry with a transverse trapping frequency of $\omega_\perp = 2\pi \times 500\text{Hz}$. The current through the sample wires has been adjusted to 180mA in each case. The atomic density has been detected by resonant absorption imaging after 1 ms time of flight. For imaging, a 50$\mu s$ pulse of linear out of plane polarized light at an intensity below 10% of the saturation intensity has been used. Employing diffraction limited optics, we achieve an optical resolution of $\sim 3\mu m$.

Crucial to a quantitative measurement is the characterization of the atom cloud. The important parameters are the temperature of the cloud, and its position above the wire. The total number of atoms is less important, as it cancels out through the potential normalization of each scan line.

The temperature has been extracted from the free expansion speed of the clouds’ Gaussian width in the transverse direction. This speed is measured by fitting a Gaussian profile to a series of at least 20 pictures acquired after an expansion time between 0.5 ms to 6 ms. The temperature is measured at 5 equally spaced positions within each scanned region. No systematic dependence of the atom temperature on the scan position was detected.

The height above the wire surface is determined from the distance between image and mirror image in an absorption picture acquired in-situ. The imaging beam was inclined by 6.5° toward the chip surface. The distance is extracted from the position of maximum correlation between
image and mirror image. As this height measurement is relative to the height level on the chip where the images are reflected, the correct absolute position was determined in the regime where both images are reflected by the surface of the wire studied. In order to avoid problems due to polarization rotation at the gold surface, linear polarized light has been used. The polarization direction was perpendicular to the plane of incidence. In this configuration, the polarization state is unchanged by the reflection at the gold surface (S9).

The transverse position of the cloud has been determined indirectly through a calibration of the bias fields. As a calibration standard the field distribution of the straight wires has been used. Two bias field coils have been adjusted such that the position of the trap center above the wire follows exactly the expected dependence from a rectangular wire. The center position above the wire has been checked separately, by checking the minimum position of the transverse trap frequency. From this procedure the center position is determined with an accuracy of approximately \( \pm 2 \mu m \). The y scale is determined to an accuracy of approximately 10%.

**Fabrication and characterization of the atom chip**

The atom chip was fabricated in a thermal evaporator by depositing gold onto a Si wafer covered by a 100nm SiO\(_2\) electrically insulating layer and with a thin Ti adhesion layer. In order to realize three very different wires in grain size and thickness, three separate processes had to be made on the same chip.

Wires A and B, in which the grain size is small, were deposited in two separate processes achieving different thicknesses, where no heating was applied. For wire B, the temperature of the cooled stage in the thermal evaporator reached 32°C, while for wire A it reached 53°C, as the deposition time was longer (in order to achieve the larger thickness). The lithography defining the wires was done by standard lift-off processes of a photoresist. For wire C, which required large grain size, the deposition temperature was fixed at 200°C, a temperature in which photoresists lose their properties. Therefore, in this case, the photoresist was put on after the deposition and the etching of the gold between the wires could not be done by lift-off, but rather
required ion beam milling. While fabricating one wire, care had to be taken to protect the other wires. Finally, a mirror layer of gold was deposited around the three wires in order to reflect the laser beams required for the Magneto-Optical Trap, the first stage of atom cooling.

The chip was characterized optically, electrically (room and liquid helium temperatures) and by a high resolution scanning electron microscope (SEM), an atomic force microscope (AFM) and a white-light interferometer (SL10). The actual experimental chip underwent only optical and room temperature resistance measurements before the experiment. The grain size was determined before the experiment by SEM measurements on a duplicate chip in order not to affect the experimental chip. To make sure that the grain size did not change during chip operation, the experimental chip also underwent SEM measurements after the experiment was done.

Edge roughness of the fabricated wire boundaries was measured to be (peak-to-peak) 40-50nm for and 20-30nm for wires A and B, respectively (wires fabricated using the lift-off technique), and 40 nm for the large grain size wire C, determined by the ion-beam milling etching technique.

The global structure of the top surface roughness of the wires was measured with a nanometer resolution using a white-light interferometer on the experimental chip after the experiment was done. Again, comparison to the duplicate chip was performed. A sequence of partially overlapping images of $105.6 \times 140.8 \, \mu m^2$ and $0.5 \mu m$ resolution were taken for each wire along a total length of about 1mm. The radial spectrum of the surface height variations (shown in Fig. 3) was determined by averaging the two-dimensional Fourier transform of the images over all wave-vector directions and over all images. The measurement noise was estimated by taking the power spectrum of the differences between pairs of overlapping measured area and averaging over all pairs. This noise was found to be similar for all three wires and was taken in Fig. 3 to be their average.

We measured also the detailed surface structure and the grain size with an AFM (again, after the experiment was done). Images of area $5 \times 5 \mu m$ (with an x-y resolution of 40nm and a
height resolution of $< 1$nm) were used for validation of the white-light interferometer data and for estimating the surface roughness at the grain size scale. Grain structure was investigated with images of area $1 \times 1 \mu\text{m}$ (with an x-y resolution of 2nm).

As for electrical measurements, as noted, the experimental chip underwent room temperature resistance measurements before the experiment. Similar measurements were also done on the duplicate chip for comparison before and after the experiment. As the experimental chip was partly broken after the experiment, low temperature resistance was measured only on the duplicate chip. Room temperature resistance measurements were performed on the central part of the wires, where the geometry of the wire was measured accurately. The low temperature measurements were done on the entire length of the wires. Concerning the two chips we note that: their room temperature resistance measurements were identical to within a few percent; their SEM and AFM analysis showed similar structure; they were made by the same processes at the same time. Hence, we assign high probability to the low temperature measurements being adequate also for the experimental chip, although strictly speaking the variance could not be measured.

### Models of current flow

#### Current flow around a circular obstacle

A cylindrical defect of radius $R$, extending over the whole thickness of a thin film, in which the conductivity is changed from its bulk value $\sigma_0$ to $\sigma_0 + \delta \sigma$, causes a deflection of the current density $J_0 \hat{x}$ impinging on the defect. If $\delta \sigma$ is negative then the current bends away from the obstacle and tends to bypass it, while if $\delta \sigma$ is positive the current is attracted toward the defect. Here we argue that this current deflection emerges from a dipole field that is created at the defect, as shown in Fig. 2A,B. This follows from Eq. (S11) which shows that a conductivity gradient is equivalent to a charge density

$$\rho = -\varepsilon_0 \frac{\nabla \sigma}{\sigma} \cdot \mathbf{E},$$

(S4)
where in our case $E = E^{(0)} \hat{x} = J_0 / \sigma_0 \hat{x}$. The charge density per unit length of the boundary at the edges of a disk of radius $R$ is then given by

$$\rho(R, \phi) = R \frac{\delta \sigma}{\sigma_0} E^{(0)} \cos \phi,$$

(S5)

where $\phi$ is the angular coordinate. Very close to the boundary an electric field is formed that points in a direction normal to the boundary and outwards from the boundary. The electric field generated everywhere by the excess charge density can be calculated by an integration

$$\delta E_j(r, \theta) = \int_0^{2\pi} d\phi \rho(R, \phi) G_j(r, R, \theta, \phi),$$

(S6)

for $j = x, y$, where the Green’s functions are given by

$$G_x(r, R, \theta, \phi) = \frac{1}{2\pi (r^2 + R^2 - 2Rr \cos(\theta - \phi))} \left( r \cos \theta - R \cos \phi \right)$$

(S7)

$$G_y(r, R, \theta, \phi) = \frac{1}{2\pi (r^2 + R^2 - 2Rr \cos(\theta - \phi))} \left( r \sin \theta - R \sin \phi \right).$$

(S8)

By performing the integration we obtain

$$\delta E(r, \theta) = \frac{\delta \sigma}{\sigma_0} E^{(0)} \times \begin{cases} \left( -1/2, 0 \right) & r < R \\ (R^2/2r^2)(\cos 2\theta, \sin 2\theta) & r > R \end{cases}$$

(S9)

To the first order in $\delta \sigma$ it follows $J_0 = \sigma_0 E^{(0)}$

$$\delta J(r, \theta) = \frac{\delta \sigma}{\sigma_0} J_0 \times \begin{cases} \left( 1/2, 0 \right) & r < R \\ (R^2/2r^2)(\cos 2\theta, \sin 2\theta) & r > R \end{cases}$$

(S10)

It can then be checked that the normal part of $\delta J$ is continuous on the boundary. The current flow inside the disk is constant, while out of the disk it has a dipole pattern whose transverse component $\delta J_y$ is proportional to $\sin 2\theta$, as seen in Fig. 2A,B.

**Current flow in a conductivity step**

Let us now consider current scattering by a conductivity step as in Fig. 2C, where the conductivity is $\sigma_0$ except in the shaded area between two parallel lines inclined by an angle $\theta$, where the conductivity is $\sigma_0 + \delta \sigma$ ($|\delta \sigma| \ll \sigma_0$). The voltage applied along the $\hat{x}$ direction induces a
constant current $J_0 \hat{x} = \sigma_0 E^{(0)} \hat{x}$ everywhere. The current continuity equation $\nabla \cdot \mathbf{J} = 0$ together with Ohm’s law $\mathbf{J} = \sigma \mathbf{E}$ give rise to the following equation for the electric field

$$\nabla \cdot \mathbf{E} = -\frac{\nabla \sigma}{\sigma} \cdot \mathbf{E}. \quad (S11)$$

This equation is equivalent to Maxwell’s equation $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$, where $\rho$ is the charge density.

It follows that the gradients of the conductivity at the right (+) and left (-) boundaries of the step are equivalent to homogeneous charge densities

$$\rho_\pm = \pm \epsilon_0 (\delta \sigma/\sigma_0) E^{(0)} \cos \theta \quad (S12)$$

per unit area. Equivalently to the field formed between the two plates of an oppositely charged capacitor, here as well an electric dipole field $\mathbf{E}^{(1)} = \hat{n} (\rho_- - \rho_+)/2\epsilon_0$ is formed between the two interfaces in a direction normal to the interfaces. This additional electric field induces an additional current $\delta \mathbf{J}$ inside the conductivity step. To first order in $\delta \sigma$ this current is given by

$$\delta \mathbf{J} = \delta \sigma \mathbf{E}^{(0)} + \sigma_0 \mathbf{E}^{(1)}$$

$$= J_0 \frac{\delta \sigma}{\sigma_0} \left[ \hat{x} - \cos \theta (\cos \theta \hat{x} + \sin \theta \hat{y}) \right]. \quad (S13)$$

The transverse current component $J_y$ is then proportional to $\cos \theta \sin \theta = \sin 2\theta/2$.

**The emergence of the macroscopic observable**

We would now like to describe how the macroscopic observable exhibiting elongated patterns of high transverse current tens of microns long emerges from the microscopic scattering in the wire estimated to have a length scale of tens of nano-meters (e.g. grain size or diffusion length). We choose to describe this microscopic-macroscopic interplay by analyzing two separate models as they are complementary in the sense that one (circular defects) begins with the microscopic description and the other (conductivity step) from the macroscopic one. First, we have shown that the transverse current distribution around a circularly symmetric conductivity defect exhibits a $\sin 2\theta$ dependence (Eq. S13). Thus, a high concentration of many such microscopic defects with the same sign of $\delta \sigma$ will form a similar macroscopic pattern, which is
characterized by elongated patterns along the $\pm 45^\circ$ direction, as seen in Fig. 1. As indicated by the spectrum for the measured surface height of Fig. 3 and the spectral analysis of Fig. 4, this distribution is correlated over length scales of tens of $\mu m$ and characterized by a spectral dependence $\delta \sigma(k) \propto 1/|k|$. Such a correlated distribution of scatterers is indeed characterized by a bunching of scatterers of the same sign in regions with a typical width of tens of $\mu m$. This allows for a constructive build up of transverse currents to occur along a macroscopic $45^\circ$ line, and hence gives rise to the macroscopic observable, as demonstrated in Figure S2.

The conductivity step model allows for a quantitative understanding of the emergence of macroscopic patterns containing elongated regions of high transverse current from a completely random conductivity inhomogeneity. Although the conductivity perturbations are random and do not have any angular preference, their $\sim 1/|k|$ spectral dependence implies that they may be expanded in a Fourier series of plane waves with predominantly large wavelengths. Each such plane wave is made of wavefronts, which are elongated regions similar to the conductivity step, with alternating sign. Hence, our macroscopic conductivity step in fact represents one Fourier component of the real space distribution of microscopic defects. The above result (Eq. S6) predicts maximum electron scattering from conductivity steps which are oriented at $\pm 45^\circ$. This explains the emergence of the elongated regions with that orientation in the maps of the transverse current distribution and gives the expression of Eq. 1, which allows a quantitative analysis of the current distribution for an arbitrary conductivity inhomogeneity distribution.

**Surface and bulk models**

Here we present some details about the models used to fit the measured power spectrum of the magnetic directional variations $\beta$ in Fig. 4. These fits are also displayed on a single axes and a linear $k_x$ scale in Fig. S3.

The surface models are based on the power spectrum of the measured top surface corrugations $\delta z_+(k)$ presented in Fig. 3. In order to fit the power spectrum of the thin wires B and C we have assumed that the bottom surface is partially correlated with the top surface and has the
form $\delta z_-(k) = \eta e^{-(k/k_0)^2} \delta z_+(k) + \delta z_{uncorr}^-(k)$, where $\delta z_{uncorr}^-$ is the part of the bottom surface which is uncorrelated with the top surface. This part is small, especially at long length scales and therefore we neglect it. The best fit to the data is obtained with $\eta = 0.999$ and $\eta = 0.9993$ for wires B and C, respectively and correspondingly $k_0 = 0.657 \mu m^{-1}$ and $1.39 \mu m^{-1}$ for the two wires. The resulting thickness variations $\delta H = \delta z_+ - \delta z_-$ are therefore much smaller than the measured top surface corrugations at the long scales $> 50 \mu m$, but become more and more significant when approaching the short length scales, such that at the length scale of few microns $\delta H$ fluctuations are of the same order of magnitude as the top surface corrugations $\delta z_+$. 

The situation is completely different for the thick wire A. In this case, a maximal contribution of thickness variations originating from a flat bottom surface $\delta z_- = 0$ or completely uncorrelated top and bottom surfaces, is not large enough to account for the measured current directional fluctuations. A fit to the data is obtained by assuming bulk conductivity inhomogeneity of the form $\delta \sigma(k) = \delta \sigma(k_0)(k_0/k)^\nu$. For the thick wire A a best fit is obtained for $\nu = 1$ ($\delta \sigma(k) \propto 1/|k|$) and $|\delta \sigma(k_0)|/\sigma_0 = 3.4 \cdot 10^{-4}$ at $k_0 = 2\pi/680 \mu m$. A model taking the maximal contribution of surface roughness (uncorrelated top and bottom surfaces) into account gives the minimal required contribution of the bulk conductivity fluctuations of $\delta \sigma_{rms}/\sigma_0 = 8.7 \cdot 10^{-4}$ (S12).

Applying a bulk model with the same spectrum for the thin wires B and C overestimates the measured data. If we ignore the surface contribution in the thin wires (assuming no thickness variations), a best fit with a bulk conductivity inhomogeneity model is obtained for both wires with $\nu = 1/2$ $\delta \sigma(k) \propto 1/|k|^{1/2}$ and $|\delta \sigma(k_0)|/\sigma_0 = 4.3 \cdot 10^{-5}$ and $2.3 \cdot 10^{-5}$ for wires B and C, respectively.

**Spectral decomposition of the measured magnetic field patterns**

The difference in the spectral composition of the magnetic field directional fluctuations above the three wires, which is evident in the maps shown in Fig. 1, may be quantitatively character-
ized in a few ways.

First, we define the average wavelength of the pattern of the magnetic field angle $\beta$ by

$$\lambda_{\beta} = \frac{2\pi}{\sqrt{\langle k^2 \rangle}}$$

where $\langle k^2 \rangle$ is an average over the two-dimensional power spectral density. When we use a cut-off of $k = 0.3 \mu m^{-1}$ we find that $\lambda_{\beta} = 77 \mu m$, $46 \mu m$ and $48 \mu m$ for wires A, B and C, respectively. This is related to the fact that the power spectral density drops faster for higher $k$ in the thick wire. The cut-off value was chosen to correspond to the beginning of the tails in the spectra presented in Fig. 4, where the tails are due to noise and imaging limitations.

The difference in the spectral decomposition is also clear from the slope of the curves in Fig. S3. The major part of the longitudinal power spectrum in Fig. S3 (for $0.05 < k_x < 0.3 \mu m^{-1}$) can be well fitted with an exponential function $P(k_x) \propto e^{-\alpha k_x}$ with $\alpha = 18$, $11$ and $9 \mu m$ for the three wires.

The physical origin of the difference between the spectral decomposition of the thick wire and the thin wires lies in one of two optional explanations. The bulk inhomogeneity model fits the power spectrum of the thick wire when $\delta \sigma(k) \propto 1/k$, while a best fit is obtained for the thin wires when $\delta \sigma(k) \propto 1/k^{1/2}$. However, there is no satisfying explanation for this difference in the spectral composition of the bulk inhomogeneity in wires with different thicknesses. A more plausible explanation is obtained from the model of partially correlated top and bottom surfaces. There is a good reason to believe that wires generated by the evaporation process of a gold layer show more correlation between the top gold surface and the bottom substrate surface when this layer is thinner. Secondly, for a given surface corrugation the relative thickness variation is inversely proportional to the average thickness of the wire, so that the contribution of conductivity variations originating from surface corrugations becomes more dominant in thin wires relative to bulk inhomogeneity variations, which are not expected to grow with reduced thickness.
Figure S1. Schematic of magnetic field measurements with an ultracold thermal cloud (top figure) and a BEC (bottom figure). The thin plots underneath each main diagram show the integrated column density as measured in an absorption image. For a thermal gas the measurement range is given by a few times the temperature ($k_B T$). For a BEC the measurement range is given by the chemical potential $\mu$. The BEC has better resolution, but a smaller measurement range given by the chemical potential. If the potential differences become too large, then there will be regions with no atoms in between, and it is not guaranteed that the chemical potential is still the same in the disjunct regions. The thermal atoms have a larger and more smooth measurement range. Even relatively high peaks in between can be scaled.
Figure S2. The emergence of macroscopic elongated regions with preferred orientation in the pattern of the magnetic field fluctuations above a wire from a distribution of microscopic circularly symmetric defects. (left) The distribution of defects with positive (red) or negative (blue) conductivity change $\delta\sigma(x, y)$ is correlated with a $1/k$ spectral dependence, but has no preferred orientation, as indicated by the angular spectrum $P(\theta)$ (bottom). (middle) Bunches of point-like defects create a current distribution having an angular spectrum (bottom) which is maximal around $\theta = \pm 45^\circ$. (right) the resultant magnetic field fluctuations at $z_0 = 3.5\mu m$ above the plane averages the fine structure of the currents by the exponential factor $e^{-kz}$ which suppresses short wavelength variations but keeps the orientational preference.
Figure S3. Power spectrum $P(k_x)$ of the magnetic field angle for the three wires with a fit using the bulk conductivity inhomogeneity model (black lines) and the surface corrugations model for the two thin wires B (blue) and C (red). It is clearly seen that the slope of $P(k_x)$ for the two thin wires is similar, while it is larger for the thick wire A. Within the framework of the bulk model this is attributed to a $1/k$ dependence of the slope of the conductivity inhomogeneity for the thick wire and $1/k^{1/2}$ for the thin wires. A plausible explanation is that both the bulk inhomogeneity and the contribution from uncorrelated top and bottom surfaces are dominant for the thick wire, while for the thin wires thickness variations are suppressed at long wavelengths by correlated top and bottom surfaces.
References and Notes


S10. The white-light interferometer used here was a Zygo New View 200, with a 50x objective. The atomic force microscope (AFM) was a Veeco AFM / SPM Dimension 3100 with a Nanoscope4 controller. For further details see www.zygo.com and www.veeco.com.


S12. This rms value is calculated from the bulk model with \( \delta \sigma(k_0 = 2\pi/680\mu m^{-1})/\sigma_0 = 2.7 \times 10^{-4} \) and cutoff at \( k = 2\pi/3\mu m^{-1} \).