Supporting Online Material for

Measurement of Universal Thermodynamic Functions for a Unitary Fermi Gas
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Measurement of universal thermodynamic functions for a unitary Fermi gas: Supporting Online Material

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Materials and Methods

Trapping potential. The trap used in this experiment consists of an optical dipole trap and a magnetic trap. The optical dipole trap is formed by a focused laser beam (λ=1064 nm), and it mainly provides radial confinement. Axial confinement of $\omega_z=2\pi\times16$ Hz is produced by the magnetic field curvature at 834 gauss.

To confirm that our results are independent of trap geometry, we measured the values of $f_\Omega[\theta]$ not only with two different ellipticities of the optical trap with beam waists of ($w_x=26$ μm, $w_y=34$ μm) and ($w_x=26$ μm, $w_y=40$ μm), but also with different aspect ratios of $\sqrt{\omega_x\omega_y}/\omega_z=70-15$, which change at different trap depths. We thus confirmed that the universal function determined by our method is independent of the trap geometry. We note that the trapping potential can be considered to be a three-dimensional harmonic potential, because the axial confinement which is realized mainly by the bias magnetic field is harmonic and also the
radial confinement implemented by the Gaussian laser light is almost harmonic for the trapped atoms because the potential is deep as compared to the Fermi energy \( V_{\text{depth}} \approx 10E_{F,\text{trap}} \).

**Thermometry.** We utilized the energy-temperature relation in thermometry for the trapped unitary Fermi gas; this relation is determined by the Duke group in a model-independent manner (S1). By limiting the energy region to \( E_{\text{total}}/E_{F,\text{trap}} < 1.3 \) \( (T/T_{F,\text{trap}} < 0.5) \), we confirmed that the two slightly different energy-temperature relation reported in (S1) make no significant difference in the final result of the universal functions. Then, we limited all our measurements (universal function, energy comparison, and sound wave) to this energy region.

**Data analysis.** Absorption images were taken perpendicular to the axial direction after a 3 ms free expansion, and these images were integrated along the radial direction to obtain the axial line density. We constructed the *in situ* three-dimensional atomic density distribution from the axial line density under the assumption of local density approximation (LDA) as follows. While a unitary Fermi gas has the exact Thomas-Fermi (TF) density profile for an ideal Fermi gas both at zero temperature and at high temperatures where the Maxwell-Boltzmann limit is approached (S2), a slight deformation of the density profile from the TF form has been observed below the critical temperature (S3). For a better fit with the density profile, we adopted a function

\[
n(z; t_{\text{TOF}}) = \frac{N}{\sqrt{\pi} \sigma_z(t_{\text{TOF}})} \cdot \frac{Li_{5/2} \left[ -\xi e^{-z^2/\sigma_z(t_{\text{TOF}})} \right]}{Li_3[-\xi]} \]

with three (usually, two are considered) fitting parameters, \( N \), \( \sigma_z(t_{\text{TOF}}) \), and \( \xi \). Here, \( Li_n[x] = \sum_{k=1}^{\infty} \frac{x^k}{k^n} \) is the \( n \)-th polylogarithm function. By the three-parameter fit, we have been able to describe the actual density profile well even below the critical temperature (S4). Figure S1A shows a density profile (black dots) with fitting results by TF fitting (green dotted line) and our three-parameter fitting (red solid line). Although both fitting functions fit the line density...
of the unitary Fermi gas well, we can see a slight difference between the two fitting results. Figure S1B shows

\[
\frac{\chi^2_{\text{TF}} - \chi^2_{\text{3-parameter}}}{\chi^2_{\text{TF}}},
\]

where \(\chi^2_{\text{TF}}\) and \(\chi^2_{\text{3-parameter}}\) are the residual sum of squares (RSS) of the TF fitting and the three-parameter fitting, respectively. It can be clearly seen that our three-parameter fitting gives less RSS particularly below the critical temperature, indicating that the density profile is deformed below the critical temperature and our fitting result describes the deformation of the profile better than the TF form. During a 3 ms free expansion, the axial size shrinks to a factor of about 0.96 without changing the shape due to hydrodynamic behavior in the presence of the magnetic curvature. In fact, we have found that the hydrodynamic expansion assumption is quite good up to around 5 ms. Therefore, the in situ axial size is given by \(\sigma_z = \sigma_z(\text{TOF})/b_H(\text{TOF})\), where the hydrodynamic expansion coefficient \(b_H(\text{TOF})\) is calculated from the trapping frequencies, magnetic curvature, and expansion time (S2, S5). By using the in situ line density \(n(z)\) and the triaxial trapping frequencies, we construct the three-dimensional atomic density distribution \(n(r)\). To solve Eq. (1) with \(n(r)\) and \(V_{\text{trap}}(r)\), we simplify the differential equation to

\[
\frac{d\rho(R)}{dR} + n(R) \cdot m \omega^2 R = 0
\]

by constructing the spherical density distribution and the spherical trapping potential according to

\[
n(R) = \frac{N}{\pi^{3/2}} \frac{Li_{3/2}\left[\frac{\rho - R^2/\pi^2}{\xi}ight]}{Li_3[-\xi]}
\]

and

\[
V_{\text{trap}}(R) = \frac{m}{2} \omega^2 R^2,
\]

where \(\omega = (\omega_x, \omega_y, \omega_z)^{1/3}\), \(\sigma = \frac{\omega}{\omega_x} \sigma_z\), and

\[
R = \sqrt{\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 / \omega}. \quad \text{The local pressure} \ p(R) \ \text{is obtained by solving this simplified equation.}
\]

Thus, the local universal function is given by \(f_E(R) = 3p(R)/2n(R)\sigma_T(R)\) as a function of the natural coordinate. The local reduced temperature \(\Theta(R) = k_B T / \sigma_T(n(R))\) is also determined from the spherical density distribution \(n(R)\) and the temperature \(T\). By relating \(f_E(R)\) to \(\Theta(R)\) at the same position, the universal function of the internal energy \(f_E[\Theta]\) is determined as a function of the local reduced temperature \(\Theta\).
**Condensate fraction.** The condensate fraction is determined by fitting the momentum distribution of the molecules formed after a rapid field sweep (S6-S8), to a bimodal distribution comprising a parabolic function and a Bose-Einstein function. In this experiment, the sweep time required to leave the strongly interacting region is 2 μs. This time scale is much shorter than the typical relaxation time of 500 μs (S3, S8). Therefore, the condensate fraction is not likely to change noticeably during the sweep.

**References and Notes**


Fig. S1. Axial line density of a unitary Fermi gas. (A) An experimentally obtained line density at $E_{\text{total}}/E_{F,\text{trap}}=0.64$ is shown by the black dots. The red solid line shows the fitting result using our three-parameter fitting. The green dashed line shows the fitting result using the TF profile for an ideal Fermi gas by using the method given in (S2). (B) Relative differences in RSS of the fitting between the TF fitting and our three-parameter fitting are plotted at various energies. The plot shows that our three-parameter fitting describes the deformation of the profile from the TF form below the critical energy. It can be clearly seen that the shape of a trapped unitary Fermi gas starts to deform below the critical energy and recovers to the TF profile at the lowest energy, as shown in (S9).