Supporting Online Material for

Primordial Gravitational Waves and Cosmology

Lawrence Krauss,* Scott Dodelson, Stephan Meyer
*To whom correspondence should be addressed. E-mail: krauss@asu.edu

Published 21 May 2010, Science 328, 989 (2010)
DOI: 10.1126/science.1179541

This PDF file includes:

SOM Text 1 and 2
Table S1
References
It is well known that in a de Sitter background all massless or light quantum fields will fluctuate with a magnitude which is proportional to the only dimensional parameter governing the expansion, the stored energy density, $\rho$. Einstein’s Equations relate this energy density to the expansion rate $H$, so that, for every scalar field $\phi_i$, for modes with wavenumber $k \approx H\langle (\delta \phi_i)^2 \rangle \approx H^2/2\pi$.

Remarkably, as first demonstrated by Grishchuk [1], when one linearizes Einstein’s equation for weak fields, then each of the two helicity states of a gravitational wave $h_i$ obeys precisely the equation for a massless, minimally coupled real scalar field, with a normalization factor of $\sqrt{16\pi G}$. Thus when the wavelength of a gravitational wave is of order $k^{-1} = H^{-1}$, the Hubble radius, it fluctuates with a variance

$$\langle (\delta h_i)^2 \rangle \approx 8GH^2.$$ 

As inflation stretches distances, this wavelength soon becomes much larger than the Hubble radius – it is driven outside the horizon – after which its amplitude remains fixed since no causal process can act over distances larger than the horizon. After inflation ends, these modes return inside the horizon as a stochastic background of gravitational waves and begin to oscillate and redshift. The resulting tensor mode power spectrum can be defined by [2, 3, 4], (for a recent review see [5])

$$\langle h_i(k)h_j(k') \rangle = (2\pi)^3 \delta_{ij}\delta^3(k + k') \frac{2\pi^2}{k^3} P_t(k)$$

Clearly, the larger the energy scale at which inflation occurs, and hence the larger the value of $H$, the larger will be the residual gravitational wave signal.

There are traditionally two different ways of framing the question of the observability of such a primordial gravitational wave spectrum. First, in the context of the Cosmic Microwave Background, one can compare the predicted power in tensor modes to the power spectrum for direct primordial energy density fluctuations. Since these scalar density fluctuations are also due to fluctuations in quantum fields, these are also proportional to $(H/2\pi)^2$. However the mechanism
by which quantum fluctuations turn into density fluctuations also depends, in the simplest inflationary models, on the details of the potential for the field $\phi$ that is driving inflation (assuming only a single field is involved), and how quickly this field exits from inflation (e.g. \cite{6,7,8,9}). The form for such scalar density perturbations in these models is:

$$P_s(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \bigg|_{k \approx H}.$$  

The quantity $r$ is defined as the ratio of these two power spectra \cite{5},

$$r \equiv \frac{P_t}{P_s} = \frac{8 \left(\frac{\dot{\phi}}{H}\right)^2}{G H}.$$  

This quantity plays a definitive role in parametrizing the detectability of gravitational waves in CMB experiments. The next generation of detectors is being designed with the intention of reaching a sensitivity to $r \geq 0.01$. Note however that the model dependence of $r$ (due to the presence of $\dot{\phi}$) is somewhat deceptive, due to the unfortunate (but conventional) use of $P_s$ in the numerator. It is worth emphasizing once more that the gravitational wave signal is however fixed by the scale of Inflation, and it is the scalar density signal that depends upon the details of inflationary model potential. Therefore, once $r$ is measured, the energy density during inflation will also be determined; the two are related via:

$$V^{1/4} = 1.06 \times 10^{16} \text{GeV} \left(\frac{r}{0.01}\right).$$  

Current limits of $r < 0.3$ already have severely constrained or ruled out a variety of single field inflationary potentials and give hope that increasing the experimental sensitivity to $r$ could yield a positive non-zero signature for gravitational waves in the near future.

At the same time, however, there is an independent argument that suggests one should be cautious before counting on such a possibility. Inflation requires at least 60 exponential e-foldings in order to resolve all the cosmological puzzles that it was designed to resolve. One can ask what the total field excursion is during this period, and the larger the scale of inflation, the larger the magnitude of the inflaton field at the end of inflation. One finds, in fact, again for single field inflation \cite{5}

$$\frac{\Delta \phi}{M_{pl}} \geq 1.06 \times \left(\frac{r}{0.01}\right)^{1/2}.$$  

2
where $M_{pl}$ is the Planck mass. Field excursions larger than the Planck scale lead one to territory where quantum gravitational effects may be significant, and hence where simple quantum field theoretical arguments become suspect. In fact, in many string theory models the value of the inflaton field is associated with an excursion in an extra dimension, which is itself restricted to be Planck length in size. In this case, it is simply impossible for the field to take values larger than the Planck scale, and one would expect, therefore that $r < .01$. More recently, however, it has been realized that string theory has enough flexibility to allow for complicated multifield models in which the relation above is altered, and $r$ can exceed 0.01 (eg. see [10]). Indeed, string theorists now claim that distinguishing between these two regimes (small and large $r$) will help define the symmetry structure of the underlying theory of nature. Thus, while the theoretical situation remains somewhat vague at the current time, $r = 0.01$ represents an important threshold.

Because inflation produces gravitational waves on all scales, including those that came within the horizon well before the current epoch, or before the CMB era, one might hope that other probes of gravitational waves with much smaller wavelengths and higher frequencies might be useful. One can explore the possible sensitivity of such direct and indirect probes of a stochastic background of gravitational waves by considering the contribution to the total energy density today, per logarithmic frequency interval, resulting from inflation. Since inflation predicts constant power in tensor modes on all scales before they come inside the horizon, the predicted contribution to the energy density at any time for horizon size modes will depend upon whether such modes enter the horizon during radiation domination or matter domination. This is simply because the expansion history is different during these epochs and gravitational wave modes will redshift by a different amount as the move from super-horizon to sub-horizon size.

Once inside the horizon, gravitational waves simply redshift like radiation. During the radiation dominated era their contribution to the total energy density therefore remains constant. However during matter domination, their contribution falls by a factor identical to that of photons. Today, this factor is given by $\rho_{rad}/\rho_{total} \approx 2 \times 10^{-5}$, given current best estimates of the Hubble constant. The contribution to $\Omega$ for modes that entered the horizon during radiation domination today will therefore be suppressed by this factor, barring further suppression due to possible reheating of the radiation gas by entropy injection since the time the gravitational wave modes entered the horizon.
SOM2: CMB Polarization experiments

Adapted from the CMBpol Mission Concept Study Group
Report to the Astro 2010 Decadal Committee on Astrophysics

Table S1: Future Suborbital CMB Polarization Experiments.

<table>
<thead>
<tr>
<th>Balloon-borne:</th>
<th>Technology</th>
<th>FWHM (arcmin)</th>
<th>Frequency (GHz)</th>
<th>Detector Pairs</th>
<th>Modulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spider [12]</td>
<td>TES</td>
<td>60/40/30</td>
<td>96/145/225</td>
<td>288/512/512</td>
<td>HWP/Scan</td>
</tr>
<tr>
<td>PIPER I</td>
<td>TES</td>
<td>21/15</td>
<td>200/270</td>
<td>2560/2560</td>
<td>VPM</td>
</tr>
<tr>
<td>PIPER II</td>
<td>TES</td>
<td>14</td>
<td>350/600</td>
<td>2560/2560</td>
<td>VPM</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ground-based:</th>
<th>Technology</th>
<th>FWHM (arcmin)</th>
<th>Frequency (GHz)</th>
<th>Detector Pairs</th>
<th>Modulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS [13]</td>
<td>TES</td>
<td>30</td>
<td>150</td>
<td>200</td>
<td>HWP</td>
</tr>
<tr>
<td>ACTpol [14]</td>
<td>TES 2.2/1.4/1.1</td>
<td>90/145/217</td>
<td>~ 1000</td>
<td>Scan</td>
<td></td>
</tr>
<tr>
<td>BICEP 2 [15]</td>
<td>TES</td>
<td>37</td>
<td>150</td>
<td>256</td>
<td>HWP/Scan</td>
</tr>
<tr>
<td>MBI/QUBIC [16]</td>
<td>NTD</td>
<td>60</td>
<td>100</td>
<td>4</td>
<td>Int.</td>
</tr>
<tr>
<td>Poincare [17]</td>
<td>TES</td>
<td>84/30/24</td>
<td>40/90/150</td>
<td>36/300/60</td>
<td>VPM</td>
</tr>
<tr>
<td>PolarBear [18]</td>
<td>TES 7/3.5/2.4</td>
<td>90/150/220</td>
<td>637</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QUIJOTE [20]</td>
<td>HEMT</td>
<td>54/24</td>
<td>10-30</td>
<td>34</td>
<td>HWP</td>
</tr>
<tr>
<td>SPIpol [21]</td>
<td>TES 1.5/1.2/1.1</td>
<td>90/150/225</td>
<td>~ 1000</td>
<td>Scan</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
Abbreviations in the modulator column are for halfwave plates (HWP), pure scanning (Scan), scanning with stepped HWP (HWP/Scan), variable-delay polarization modulators (VPM), waveguide phase switch ($\phi$-switch) and interferometers (Int.); experiments with no hardware polarization modulator are indicated by a dash, and will reconstruct polarization via their scan modulation only.

Table S1 lists current and future efforts to search for B-mode polarization. The list illustrates that a variety of technologies and observing strategies are being used. Observations over a wide range of wavelength bands is used to separate galactic from cosmic signals. A range of different angular resolutions, optimized for different, but in many cases overlapping, science goals. The experiments employ a variety of means to detect the polarization. This leads to vastly different implementation of control of systematic errors. This multiplicity of techniques and approaches should be vigorously supported over the next decade because only through experimentation can we learn how to make the measurements more robust and because the different approaches provide a crucial cross-check on the results.

This information along with further details of CMB polarization experiments as well as the technology being developed for the experiments can be found in the Report to the Astro 2010 Decadal Committee on Astrophysics.

References


