Supporting Online Material for

Topography of the Northern Hemisphere of Mercury from MESSENGER Laser Altimetry


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This PDF file includes:
Materials and Methods
Table S1
Figs. S1 and S2
References
We performed a spherical harmonic expansion of northern hemisphere planetary radii, or shape, derived from the MLA observations by least-squares inversion of 169,072 binned measurements binned at 0.25° x 0.25° in latitude and longitude, weighted by the cosine of latitude and constrained by a Kaula law (30) to limit amplification of noise in parts of the planet for which ranging coverage is lacking. The spherical harmonic model of the departure $H$ of the planet’s shape from a spherical surface has the form

$$H(\lambda, \phi) = \sum_{l=1}^{N} \sum_{m=0}^{l} \bar{P}_{lm}(\sin \phi) \left( \bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda \right)$$

where $\phi$ and $\lambda$ are the planetocentric latitude and longitude of the surface, $\bar{P}$ are the normalized associated Legendre functions of degree $l$ and azimuthal order $m$, $\bar{C}$ and $\bar{S}$ are the normalized spherical harmonic coefficients and are given in meters, and $N$ is the maximum degree and represents the resolution of the field. The $C$ and $S$ coefficients provide information on the distribution and amplitude of topography. Because of the absence of data in most of the southern hemisphere, this solution is valid only in the northern hemisphere. Also shown are the orientation of the best-fit ellipse with respect to 0° longitude and the offset of the planet’s center of figure from the center of mass in the equatorial plane. Precise values of these parameters are given in Table 1 of the main text.

Coefficients of the harmonic expansion of shape to degree and order 3 are listed in Table S1, and the square root of the variance along with the formal uncertainty of the solution are shown versus harmonic degree in Fig. S1. The error spectrum is derived from the formal standard deviations of the harmonic coefficients, under the assumption that each quarter-degree median-binned value has an uncertainty of ~250 m. The weight assigned is the root mean squared residual misfit of the degree-and-order-96 model to the 169,072 quarter-degree binned values and represents mainly the uncertainty due to inadequate sampling. The harmonic degree at which the error spectrum crosses the variance curve is generally taken as the resolution of the field. The field is globally resolved only to degree and order 3 because of the absence of southern hemisphere data, but areas of the northern hemisphere are resolved to much higher degree and order.

A representation of the elliptical shape of Mercury’s equator (relative to a sphere of radius 2440 km) determined from MLA observations within the latitude band ±15° is shown in Fig. S2.
Table S1. Shape of the northern hemisphere of Mercury versus spherical harmonic degree $l$ and order $m$, in km.

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<th>$m$</th>
<th>$C$</th>
<th>$S$</th>
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Figure Captions

**Figure S1.** Square root of the power in the spherical harmonic expansion of northern hemisphere shape versus degree (red dots connected by black lines) along with the associated error spectrum (blue line).

**Figure S2.** Shape of Mercury’s equatorial region (viewed from the north) from a spherical harmonic expansion of observations acquired by the Mercury Laser Altimeter. The dashed lines show the interquartile scale of mean radius (in km) over the latitude band ±15° within bins of width 5° in longitude; half the measurements within each longitudinal bin fall within the dashed lines, providing an estimate of the uncertainty in the equatorial shape. The red solid line connects the points on the equator at which the spherical harmonic fit to shape (degrees 0 to 2) reaches a maximum radius. The red dot is the degree-1 term in the spherical harmonic model and corresponds to the offset of Mercury’s center of figure from its center of mass (black dot) in the equatorial plane for this model.
References and Notes


3. The MLA is a time-of-flight laser rangefinder that uses direct detection and pulse-edge timing to determine precisely the range from the MESSENGER spacecraft to Mercury’s surface. MLA’s laser transmitter emits 5-ns-wide pulses at an 8-Hz rate with 20 mJ of energy at a wavelength of 1064 nm. Return echoes are collected by an array of four refractive telescopes and are detected with a single silicon avalanche photodiode detector. The timing of laser pulses is measured with a set of time-to-digital converters and counters and a crystal oscillator operating at a frequency that is monitored periodically from Earth.


12. See supplementary material on *Science* Online.


15. Median differential slope (29) removes the effect of larger-scale tilts from the elevation difference at a given baseline length. At each location $x$, half the difference in elevations between points one baseline length ahead and one behind along the altimetry ground track is subtracted from the difference in elevation between points one-half baseline length ahead and one-half behind, as $\tan \alpha = \frac{h(x + l/2) - h(x - l/2) - 0.5(h(x + l) - h(x - l))}{l}$, where $h$ is elevation, $l$ is the baseline length, and $\alpha$ is the differential slope.


