Supplementary Materials for

Conduction of Ultracold Fermions Through a Mesoscopic Channel
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Materials and Methods
Fig. S1
Effective local chemical potential for the diffusive case

In order to extract local transport quantities, we introduce an effective local chemical potential. To this end, consider a small region of the channel around position $y$. The line density around this point at equilibrium is $n_l(y)$ and the line density in the presence of current is $n_l(y) + \tilde{n}_l(y)$.

Since there is no interparticle scattering at the length scale of that small region, the energy distribution in the presence of a current is not given by the equilibrium (Fermi-Dirac) distribution. Indeed, the density varies at the short scale of the scattering with impurities or with the external potential, while the distribution relaxes towards equilibrium on the scale of the interparticle interaction mean free path. Following a standard procedure in mesoscopic physics (see (25), chap.2 part 3), we attribute to the region around $y$ an effective chemical potential $\mu(y)$, which gives the observed density when inserted in the Fermi distribution, and coincides with the chemical potential in the reservoirs. With a current flowing between the reservoirs, the atoms are not at thermal equilibrium, and the effective chemical potential does not characterize the energy distribution of atoms. It is only understood as the energy scale associated with the local density.

We now consider the relation between $\mu$, $\tilde{n}_l$ and the line compressibility at equilibrium $\kappa_l = \partial n_l / \partial \mu$. Let $f(\mu, T, E)$ be the Fermi-Dirac distribution at temperature $T$, chemical potential $\mu$ and energy $E$, and $g(E)$ be the density of states.

\[
\tilde{n}_l = \int f(\mu, T, E) - f(\mu_0, T, E) g(E) dE = (\mu - \mu_0) \frac{\partial}{\partial \mu} \int f(\mu_0, T, E) g(E) dE = (\mu - \mu_0) \frac{\partial n_l}{\partial \mu} = \tilde{\mu} \kappa_l, \tag{S1}
\]

where we have introduced the local chemical potential at equilibrium $\mu_0$, which follows from the local density approximation at equilibrium, and $\tilde{\mu} = \mu - \mu_0$ the local deviation of the chemical potential away from equilibrium.

Local resistivity for the diffusive case

Knowing the current $I$ flowing through the channel, we now introduce a local resistivity

\[
\rho(y) = \frac{\partial \tilde{\mu}}{\partial y} \frac{1}{I}, \tag{S5}
\]

which reflects the local scattering with obstacles. Although the Joule heating happens deep in the reservoirs where excited atoms release their energy via collisions with other atoms, the momentum randomization due to scattering with impurities is local, and leads to a local increase of entropy as information on the direction of motion of particles is lost.

Experimentally, to get the derivative $\partial \tilde{n}_l / \partial y$, we fit a line to the line-density difference at each position $y$ within a window of $\pm 9 \mu m$ around this point.

To extract $\kappa_l$ at position $y$ we make use of the isotropy of the compressibility. We consider at fixed position $y$ the variation of the column density $n_{col}$ along the $x$ direction and compare it with the known trap shape (29). Consider
The gas around position $y$:

$$\kappa(y) = \frac{\partial n_i(y)}{\partial \mu}$$ (S6)

$$= \frac{\partial}{\partial \mu} \int n(\mu(x, y, z)) \, dx \, dz$$ (S7)

$$= \int \frac{\partial n}{\partial \mu}(x, y, z) \, dx \, dz$$ (S8)

$$= \int \frac{\partial n}{\partial x} \left( \frac{\partial \mu}{\partial x} \right)^{-1} \, dx \, dz$$ (S9)

$$= - \int \frac{\partial n}{\partial x} \left( \frac{\partial V}{\partial x} \right)^{-1} \, dx \, dz$$ (S10)

The confinement along the $x$ direction is ensured by the optical dipole trap, and is constant over the channel. Close to the center of the channel, the confinement along $z$ is mainly ensured by the two repulsive lobes of the laser beam. Therefore, the variations of the potential along the $z$ direction is independent of the variations along the $x$ direction and can therefore be taken out of the $z$ integration.

$$\kappa(y) = - \int \left( \frac{\partial V}{\partial x} \right)^{-1} \left( \int \frac{\partial n}{\partial x} \, dz \right) \, dx$$ (S11)

$$= - \int \left( \frac{\partial V}{\partial x} \right)^{-1} \frac{\partial n_{col}}{\partial x} \, dx.$$ (S12)

A slice of $\pm 9 \mu m$ around one position $y$ is taken from the density picture at equilibrium and the average along the $y$ direction is calculated. This results in a one-dimensional density profile along the $x$ direction. We fit a gaussian to such a profile and use it in Equation S12 in order to avoid the noise generated by numerical differentiations and ratios of those (29). This fits the shape of the cloud within the errorbars. In Equation S12 $V$ is taken to be the known gaussian shaped trapping potential.

**Local mobility for the diffusive case**

In electric conduction the mobility relates the drift velocity $v_d$ to a potential gradient (electric field). In our case the gradient in the chemical potential is driving the current, and the atomic mobility is

$$v_d \cdot \left( \frac{\partial \mu}{\partial y} \right)^{-1} = v_d \cdot \kappa_1 \left( \frac{\partial n_i}{\partial y} \right)^{-1},$$ (S13)

where we used Equation S4.

The formalism described above can be applied to the diffusive case, where resistivities and mobilities are well defined. Extending it to the ballistic case would lead to the conclusion that the resistivity is zero and the mobility infinite at the center of the channel. The resistance observed in that case would then appear localized at the contacts with the reservoirs (25). While it is clear that the resistance originates in the contact, no dissipative process is actually localized at the contacts themselves. The energy relaxation is taking place inside the reservoirs and has a non-local character.

**Properties of the confining potential**

The channel is imprinted on the atoms using a 532 nm wavelength laser beam, with waists 30.2(3) and 10.3(3)$\mu m$ along the $y$ and $z$ directions, respectively. This beam passes through a holographic plate (Silios) dephasing the upper part of the beam by $\pi$ with respect to the lower part (21, 22). The beam has been characterized in a test setup and shows a slight asymmetry along the $y$ axis away from the center. The contrast of the central region of the beam compared to the lobes is larger than 0.99. In the parameter regime explored in the paper, the residual light leads to a repulsive potential along the $y$-axis smaller than 40 nK, much smaller than the oscillation frequency along $z$. 
The oscillation frequency along the tightly confining direction $z$ has been measured in-situ using parametric heating on a microscopic cloud (20), and the measured frequency agrees with the fitted curvature of the intensity profile observed on the test setup. Along the propagation direction of the beam, the measured curvature of the intensity profile varies by less than 5% on a length scale of 200 $\mu$m, and is therefore constant over the transverse radius of the cloud.

Properties of the disordered potential

The disordered potential is created by passing through a light diffuser (Luminit) with a beam at 532 nm, then through a high resolution microscope objective (20), corrected for aberrations at that wavelength. The resulting disordered potential is observed directly in-situ using a second microscope identical to the first. In this way, we directly characterize the potential correlation properties by taking pictures of the potential as seen by the atoms in the glass cell. Figure S1 presents a typical observation of the potential characteristics. Figure S1 A shows a zoom on the central part of the disordered potential where correlation properties are computed. Figure S1 B shows the measured correlation function obtained using the inverse Fourier transform of the power spectrum of A. Figure S1 C shows a cut along the horizontal axis, of Figure B, together with a Gaussian fit, yielding an observed correlation radius ($1/\sqrt{e}$ radius) in this direction of 460 nm. In the other direction, the same fit yields an observed correlation radius of 560 nm. To obtain a faithful estimate of the correlation properties of the disorder on the atoms, we deconvolve the correlation properties using the measured, gaussian point spread function of the microscope used to image the pattern (20). This yields a correlation radius averaged for the two directions of 370 nm. In addition, using the in-situ images of the light intensity, we also observed the expected exponential probability distribution of intensities.

![Intensity correlations](image)

FIG. S1: Properties of the disordered potential. (A) Central region of the disordered potential as observed with the high-resolution microscope used to image the atoms. The total width of the picture is 30 $\mu$m. (B) Correlation function of the intensity distribution observed in A. (C) Horizontal cut of B, showing the correlation properties of the disordered potential (in blue). The solid green line shows a Gaussian fit yielding a measured correlation radius of 460 nm.

The envelope of the disorder potential is straightforwardly obtained from a gaussian fit of the observed profile. The total power sent through the microscope, together with the average fitted profile of the envelope, yields the average depth of the disordered potential cited in the text.