Supplementary Materials for

Quantum Limit of Heat Flow Across a Single Electronic Channel

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Materials and Methods

1 Sample and measurement setup

Sample. The sample is nanostructured by standard e-beam lithography in a 94 nm deep GaAs/Ga(Al)As two-dimensional electron gas of density $2.5 \times 10^{15} \text{ m}^{-2}$ and mobility $55 \text{ m}^{2}\text{V}^{-1}\text{s}^{-1}$. The ohmic contacts to the buried two-dimensional electron gas are obtained with a standard technique: a metallic multilayer of nickel-gold-germanium is diffused into the semiconductor by heating the sample.

Cryogenic environment. The measurements were performed in a dilution refrigerator. At low temperature, the electrical lines were carefully filtered and thermalized by inserting long ($0.3 - 1 \text{ m}$) and resistive ($300 \Omega/\text{m}$) wires into 260 $\mu$m inner diameter CuNi tubes. The sample was further protected from spurious high energy photons by two shields, both at the mixing chamber temperature.

Low-frequency polarization and measurement setup. The sample was connected in a multi-terminal configuration with cold grounds, which is typical for samples in the quantum Hall regime (see article Fig. 1B). It was current biased with a $16 \text{ M} \Omega$ polarization resistor located at the 4.2 K stage of the dilution refrigerator. Note that the current biasing is performed through a distinct experimental line and sample ohmic contact (see article Fig. 1B). Note also that such current biasing is immune to the thermoelectric voltages developing along the lines between room and base temperature (this can be checked directly on supplementary Fig. S4 from the very precise location of the dip near zero bias current). The electrical conductances are obtained by three point measurements, using standard lock-in techniques at frequencies below 100 Hz. The polarization/measured currents are converted on-chip into voltages by taking advantage of the well defined quantum Hall resistance to ground.

Injected Joule power in the micron-sized ohmic contact. The applied DC current $I_{\text{DC}}$ (see article Fig. 1B) is converted on-chip through the well-defined quantum Hall resistance into a DC voltage $V_{\text{DC}} = I_{\text{DC}}/\nu G_e$. Using the standard Landauer-Büttiker scattering formalism (34), we find that the Joule power dissipated into the micron-sized ohmic contact reads:

$$J_Q = \frac{1}{2} \frac{G_e V_{\text{DC}}^2}{1/n_1 + 1/n_2}, \quad (S1)$$

with $(1/n_1 + 1/n_2)/G_e$ the two terminal resistance across the QPCs and micron-sized ohmic contact. The factor one half with respect to the standard two terminal expression for the total Joule power $V_{\text{DC}}^2/R$ results from the equal power dissipated into the cold electrodes at temperature $T_0$. Note that there are no such complications as Peltier or thermopower effects for perfectly transmitted channels (see e.g. (8,22) and the supplementary reference (35)).
2 Noise measurement setup

A simplified circuit description of the noise measurement apparatus is displayed in supplementary Fig. S1A, and the practical implementation is shown in supplementary Fig. S1B.

Amplification chain. The measurements are performed with an ultra-low noise preamplifier based on a home-grown high electron mobility transistor (24) thermalized to the 4.2 K stage of the dilution refrigerator. A simplified schematic of the cryogenic preamplifier is shown in supplementary Fig. S1B. It is tuned with the DC voltages \( V_{DD} \) and \( V_S \), while the gate port of the HEMT is DC-grounded through the superconducting coil (see supplementary Fig. S1B). In the investigated configurations, the cryogenic preamplifier input noise was \( \sim 0.2 \text{nV}/\sqrt{\text{Hz}} \) (see supplementary Fig. S3D for the measured input noise spectral density of the full amplification chain) and its voltage gain was \( \sim 5 \).

After a second stage of amplification at room temperature (amplifier NF SA-220F5), the signal is digitized at 10 MS/s (NI PXI-5922) to compute the noise spectral density. The noise spectral density is then integrated around the resonance over a finite bandwidth, which is tuned separately for each filling factor in order to optimize the signal-to-noise ratio.

Resonant circuit. A resonant \( L/C \) tank (see supplementary Fig. S1) is used to shift the working frequencies in the MHz range (near 0.7 MHz), where the cryogenic preamplifier shows the best performance. Another motivation is that many extrinsic noise sources become negligible in this frequency range, such as low-frequency charges trapping in the substrate or vibration noise.

The inductance \( L \) is realized by a superconducting coil thermalized to the mixing chamber of the dilution refrigerator. The parallel capacitance \( C \) is mostly due to the distributed capacitance along the coaxial lines connecting the sample to the cryogenic preamplifier. To model the dissipation between the output of the sample and the input of the amplifier we introduced a parallel resistance \( R_p \) to the tank. Here the dissipation mostly results from the resistive coaxial wire between the sample and the superconducting coil (see supplementary Fig. S1B). The precise check of this simplified model for the resonant circuit and the determination of the resonant circuit parameters is detailed below, in the corresponding subsection of "Calibrations" \( (R_p \sim 94.5 \pm 2 \text{k}\Omega \text{ for amplification chain 1, behind QPC}_1 \text{ and } R_p \sim 73.7 \pm 1.5 \text{k}\Omega \text{ for amplification chain 2, behind QPC}_2) \). The 4.7 nF capacitance between sample and inductor (see supplementary Fig. S1B) blocks the DC current along the measurement line and isolates the DC voltage at the preamplifier gate port from the sample bias voltage.

3 Calibrations

Amplification chain. The overall amplification chain (including both cryogenic and room temperature preamplifiers) is calibrated from the temperature dependent Johnson-Nyquist noise
down to 50 mK following e.g. (36), see supplementary Fig. S2. The calibration was repeated for each filling factor \( \nu \in \{3, 4\} \), each corresponding to a different sample impedance \( h/\nu e^2 \) at the input of the preamplifier and to a different choice of bandwidth \( \delta f \) for the integration of the measured noise spectral density. The integrated noise measured at the output of the full amplification chain (at the output of the room temperature amplifier) is plotted in supplementary Fig. S2 as a function of temperature for the two filling factors \( \nu = 3 \) and \( \nu = 4 \).

Note that the calibrated overall gain matches the less accurate procedure using the known gain of the room temperature amplifiers (\( \sim 200 \)) and the cryogenic preamplifiers gain (\( \sim 5 \)) obtained by the in-situ measurements of its DC transconductance and output impedance (24). Note also that the RuO\(_2\) temperature sensor is located outside the magnet bore. It is therefore subjected to a much smaller magnetic stray field than the sample.

Subsequently, we checked during the measurements the stability of the overall amplification chain gain and noise offset. The gain stability is monitored with a fixed AC signal slightly outside the frequency range used for the noise measurement (seen as high and narrow spikes in supplementary Fig. S3A). The overall system noise is monitored from the noise signal at much higher frequencies, where the \( R_p/L/C \) tank is essentially a short circuit. In practice, we found that the amplification chains remained stable at the experimental accuracy.

**Base sample temperature.** We take advantage of the overall amplification chain calibration presented above to precisely extract the base temperature on the sample \( T_0 \) at the lowest temperatures. To do so, we measure the Johnson-Nyquist noise of the sample below 50 mK and use the known amplification gain to infer the temperature. We find that our RuO\(_2\) thermometer, directly attached to the mixing chamber of the dilution refrigerator, deviates from the noise thermometer at the lowest temperatures by a relatively small amount, smaller than 5 mK (see supplementary Fig. S2). There are several possible explanations for this relatively small discrepancy. One is the calibration of the RuO\(_2\) thermometer, which is not accurate below 40 mK. Another is that the sample itself is slightly warmer than the mixing chamber, for instance due to spurious power sources. Note however that the temperature extracted from the noise measurement remains the pertinent electronic temperature. Note also that the base temperature \( T_0 \) was extracted separately at each filling factor \( \nu \).

**Resonant circuit.** Here we present a full characterization of the measurement circuit, upstream of the cryogenic amplifier. We demonstrate that this resonant circuit is accurately described by the \( R_p/L/C \) resonator depicted in supplementary Fig. S1A, and determine the value of these parameters. For this purpose, the full frequency dependent voltage noise was measured at different temperatures and for different values of the sample resistance, at \( \nu = 2, 3 \) and 4.

The symbols in supplementary Fig. S3A represent the equilibrium voltage noise spectrum \( S_V(f) \) measured at the output of the amplification chain for several temperatures and at filling factor \( \nu = 3 \). At equilibrium, the measured voltage noise is given by the Johnson-Nyquist noise of the resonant circuit in parallel to the sample, and by the noise added by the amplifiers \( S_V^{(amp)} \)
(mainly due to the cryogenic preamplifier):

\[
S_V(f) = G^2 \left( 4k_B T \text{Re} [Z(f)] + S_V^{(\text{amp})}(f) \right),
\]  

(S2)

where \(G\) is the total gain of the amplification chain and \(Z(f)\) is the measurement impedance \((\hbar/e^2)/R_p/L/C\) in the model depicted in supplementary Fig. S1A.

In order to accurately characterize the resonant circuit, one needs to cancel out the amplifier contribution \(S_V^{(\text{amp})}\). This can be done by changing the temperature, since only the resonant circuit noise contribution depends on temperature.

As illustrated in supplementary Fig. S3B for five different frequencies, we find as expected that the measured output noise spectrum is a linear function of the temperature. We perform a linear fit of the temperature dependence at each frequency of the spectrum. The obtained slopes are then plotted as a function of the frequency in supplementary Fig. S3C, together with the slopes similarly extracted at filling factors \(\nu = 2\) and 4. A simultaneous fit of those three datasets (shown as continuous lines in supplementary Fig. S3C) unambiguously yields the resonant circuit parameters \(R_p, L, C\) shown in supplementary Fig. S1A, as well as the overall gain \(G\). The high quality of the fits over a broad frequency range validates unambiguously the simple \(R_p/L/C\) model of the resonant circuit.

Interestingly, the amplifier noise \(S_V^{(\text{amp})}(f)\) can be extracted from the \(T = 0\) intercepts of the linear fits. The results are shown in supplementary Fig. S3D for the three filling factors \(\nu = 2, 3\) and 4. The noise at resonance depends on the filling factor because of the small contribution of the preamplifier current noise.

4 Expression of the noise spectral density with an increased \(T_{\Omega}\).

We now derive the relation used to extract \(T_{\Omega}\) from the measured noise spectral density (article Eq. 3). We follow a standard approach using the Landauer-Büttiker scattering description (see e.g. (27) and the supplementary references (37–39)). An identical result for the excess current noise was obtained in (28).

The current fluctuations \(\delta I_i\) emitted by the floating micron-sized ohmic contact across one of the \(n\) open electronic channels, labeled \(i\), is the sum of two distinct contributions:

\[
\delta I_i = \delta I_i^{T_{\Omega}} + \delta V_{\Omega} G_e.
\]  

(S3)

The first contribution \(\delta I_i^{T_{\Omega}}\) corresponds to the Fermi distribution of the current carrying outgoing states population, whose associated noise spectral density reads \(<(\delta I_i^{T_{\Omega}})^2> = 2k_B T_{\Omega} G_e\). The second contribution \(\delta V_{\Omega} G_e\) corresponds to the voltage fluctuations of the micron-sized ohmic contact. Since in the present sample the capacitance to ground of the floating micron-sized ohmic contact is very small \((C \approx 2.3 \text{ fF from numerical computation and dynamical Coulomb blockade measurements on the same sample, see (26); this capacitance corresponds to a parallel impedance of approximately } 100 \text{ M\Omega in the investigated frequency range})\), the
overall incoming and outgoing currents must match:

\[ n \delta V_{\Omega} G_e + \sum_{i=1}^{n} \delta I_{T_i}^{T_0} = \sum_{i=1}^{n} \delta I_{T_i}^{T_0}, \]  

(S4)

where \( \delta I_{T_i}^{T_0} \) is the incoming current fluctuation at the micron-sized ohmic contact, which was transmitted across the open electronic channel \( i \) and emitted from a large cold electrode at \( T_0 \) (\( <(\delta I_{T_i}^{T_0})^2> = 2k_B T_0 G_e \)).

Summing up the contributions of the \( n_1 \) (\( n_2 \)) open electronic channels across QPC\(_1\) (QPC\(_2\)) gives the spectral density of the emitted current noise toward the measurement electrode behind QPC\(_1\) (QPC\(_2\)). The overall current noise spectral density is found independent of the measurement electrode and simply given by:

\[ S_I = \frac{2k_B(T_\Omega - T_0)G_e}{1/n_1 + 1/n_2} + 4k_BT_0 G_e \nu. \]  

(S5)

5 Test of the noise measurement setup: comparison of excess noise from the two amplification chains.

This comparison demonstrates the relative precision of our calibration procedure, and validates a prediction of supplementary Eq. S5, namely that the noise spectral density is identical at the two measurement electrodes, independently of the number of open channels in QPC\(_1\) and QPC\(_2\).

Two separate amplification chains are used to measure the noise, each connected to one of the two measurement electrodes (see article Fig. 1B). Chain 1 is connected to the measurement electrode located behind QPC\(_1\), and chain 2 to the measurement electrode behind QPC\(_2\). According to theory, the excess noise is the same at either output, and is given by article Eq. 3. The data measured with the two amplification chains agree within less than 2%, as illustrated in supplementary Fig. S4.

6 The quasi-equilibrium hypothesis for electrons in the heated-up metal plate.

We here estimate the dwell time of an electron in the micron-size ohmic contact and compare it to the typical electron-electron interaction time. As a result of the very large electronic density of states in the metallic micron-size ohmic contact, the dwell time \( t_{dwell} \) is found in the range of 10 \( \mu s \), very much larger than the typical electron-electron interaction time, which is in the 10 ns range at low temperatures (see e.g. (30) for the related measurement of \( \tau_e \) in gold). This firmly establishes the quasi-equilibrium hypothesis that the electrons energy distribution in the micron-size ohmic contact is a hot Fermi distribution characterized by a temperature \( T_\Omega \).
The electronic dwell time $t_{\text{dwell}}$ inside the micron-size ohmic contact is evaluated from the classic expression $t_{\text{dwell}} = \nu_F \Omega h/n$ (see e.g. (40)), with $\Omega$ the volume of the micron-size ohmic contact and $\nu_F$ the electronic density of states per unit volume and energy. Injecting the volume of the micron-size ohmic contact $\Omega \approx 2 \times 10^{-3} m^3$ and a typical density of states for metals $\nu_F \approx 10^{47} J^{-1} m^{-3}$ (in gold $\nu_F \approx 1.14 \times 10^{47} J^{-1} m^{-3}$), we find $t_{\text{dwell}} \approx 60 \text{ ns}$.

7 Electron-phonon heat flow.

We now discuss the assumptions made in the model dependent analysis of the $n = 4$ reference data. In this analysis, we assume the following standard expression for the electron-phonon heat flow:

$$J_{\text{e-\text{ph}}} Q(T_\Omega, T_0) = \Sigma \Omega (T_\Omega^5 - T_0^5).$$

Note that our main result summarized by article Eq. 4 does not rely on this model (or any other), since we extracted separately the electronic heat flow by changing the number $n$ of open electronic channels.

Cold phonon hypothesis. The above expression assumes that the temperature of the phonons coupled to the electrons in the micron-sized ohmic contact is the same as the measured electronic temperature $T_0$.

Although this is a standard hypothesis for such low levels of injected power in similar devices at low temperatures (see e.g. (31) and references therein), we confirmed its validity in the present experimental setup by extracting the heat flow at different values of the temperature $T_0$: As pointed out in the article, we obtain the same value for the electron-phonon prefactor $\Sigma \Omega$ when fitting the reference $n = 4$ data measured either at $T_0 = 24 \text{ mK}$ or $T_0 = 40 \text{ mK}$. This shows that the phonons are not significantly heated up. Indeed, an actual phonon temperature significantly higher than $T_0$ would lead to different values of $\Sigma \Omega$ as the base temperature is changed. Note that this is a commonly used criterion to demonstrate the presence of hot phonons (41).

Temperature exponent. The above standard expression for the electron-phonon heat flow was verified in most experiments involving similar metal devices at low temperatures (see e.g. (31) and references therein). Nevertheless, deviations to the $T^5$ power law were also observed and predicted, for instance in highly disordered metals (see e.g. (42) and references therein). These deviations usually take the form of a different temperature power law $T^p$, where the exponent $p$ varies between 3 and 6.

In the article we focus on the low injected power regime $J_Q \lesssim 20 \text{ fW}$, where electronic heat flow is dominant, and we assume an exponent $p = 5$. We tested the validity of this hypothesis by fitting the same data using the temperature exponent $p$ as a free parameter. In order to focus on the electron-phonon coupling, which becomes dominant at high injected power, the fit is performed up to the largest injected power $J_Q \sim 90 \text{ fW}$. We found that the best fit is obtained for the exponent $p = 4.8$ (both for the $\nu = 3$ and $\nu = 4$ reference data), with a fitted electronic heat flow $\alpha_4$ found 1% higher at $\nu = 4$ (11% lower at $\nu = 3$) than the predicted quantum limit.
This confirms that the electron-phonon heat flow follows closely the above standard expression.

8 Experimental uncertainty on the quantum limit of heat flow.

Here we provide further details on the estimation of the uncertainties regarding the experimentally determined quantum limit of heat flow across one electronic channel (the uncertainties displayed article Eq. 4 and in the paragraph below article Eq. 4).

We opted for a very straightforward approach. Taking advantage of the fact that we have several different measurements of this quantum limit, corresponding to different experimental configurations, including the different investigated filling factors $\nu = 3$ and $\nu = 4$, we calculated for this ensemble of measurements not only the mean value but also the standard error on this mean value, which is the displayed uncertainty. The limitation of such an approach is that it ignores systematic errors, and in particular the error on the gain of the amplification chain. Note however that we found that the two amplification chains, calibrated separately, agree within less than 2% (see supplementary Fig. S4).

In the model-free approach, this analysis is performed on 6 different data points $\{\alpha'_{n-4}/(n-4)\}$ (see supplementary Table S1, we recall that $\alpha'_{n-4} \equiv (n-4)J_Q^c(T_\Omega, T_0)/(T_\Omega^2 - T_0^2)$), each weighted by the number of probed electronic channels $|n-4|$. This gives:

$$
\frac{J_Q^c(T_\Omega, T_0)}{T_\Omega^2 - T_0^2} = (1.06 \pm 0.07) \times \frac{\pi^2 k_B^2}{6h}.
$$

(S6)

In the model-dependent approach, this analysis is performed on 8 different data points $\{\alpha_n/n\}$ (see supplementary Table S1, we recall that $\alpha_n \equiv \alpha'_{n-4} + \alpha_4$ with $\alpha_4 \equiv 4J_Q^c(T_\Omega, T_0)/(T_\Omega^2 - T_0^2)$), each weighted by the number $n$ of open electronic channels. This gives:

$$
\frac{J_Q^c(T_\Omega, T_0)}{T_\Omega^2 - T_0^2} = (0.98 \pm 0.02) \times \frac{\pi^2 k_B^2}{6h}.
$$

(S7)

9 Supplementary data

QPC tuning. The QPCs are tuned to a set of fully open channels by adjusting the bias voltage applied to the corresponding metal split gates. We show in supplementary Fig. S5 typical conductance traces of the two QPCs measured at the filling factor $\nu = 4$. These traces display clear conductance plateaus located at integer multiples of $e^2/h$.

Comparison of symmetric sample configurations. In the article, the data for symmetric sample configurations (such as $(n_1 = 1, n_2 = 2)$ and $(n_1 = 2, n_2 = 1)$, or $(n_1 = 2, n_2 = 3)$ and $(n_1 = 3, n_2 = 2)$) are averaged in order to improve the signal to noise ratio. In supplementary Fig. S6, we show that the increase $\Delta S_I$ in the measured spectral density of the current noise is identical, at our experimental accuracy, for symmetric sample configurations, as expected from
article Eq. 3 and supplementary Eq. S5.

**Measured \( T_\Omega \) on the full span of injected power \( J_Q \).** The data shown in the article are restricted to the low injected power regime \( J_Q \lesssim 20 \, \text{fW} \), where the contribution of the electronic channels to the overall outgoing heat flow is dominant. In the main panel of supplementary Fig. S7, we display as symbols the same data obtained at \( \nu = 3 \) and \( T_0 = 24 \, \text{mK} \) and showed in the main panel of article Fig. 2, but up to the maximum injected power \( J_Q \simeq 90 \, \text{fW} \). Note that these data correspond to the raw noise measurements shown in supplementary Fig. S4. Up to the largest injected powers, electron-phonon heat transfers remain well accounted for by the same coupling parameters obtained by fitting the data at \( J_Q < 20 \, \text{fW} \) (see main text). The continuous line in supplementary Fig. S7 is calculated with the fit parameters obtained at low injected power for the \( n = 4 \) reference configuration. The dashed lines correspond to the expected contribution of the electronic heat flow \( nJ_Q^e \) in absence of electron-phonon cooling.

**Supplementary noise data for \( \nu = 4 \) and \( T_0 = 22.5 \, \text{mK} \).** We show in supplementary Fig. S8 the noise data used to extract the electronic heat current displayed in article Fig. 3B for \( \nu = 4 \) and \( T_0 = 22.5 \, \text{mK} \). The fit of the \( n = 4 \) reference configuration (continuous line) yields the value \( \Sigma \Omega = 6.5 \, \text{nW/K}^5 \) for the electron-phonon coupling and \( \alpha_4 = 4.2 \) for the electronic heat flow. The data for \( n = 3 \) is an average over the two identical configurations \((n_1 = 1, n_2 = 2)\) and \((2, 1)\); similarly, the \( n = 5 \) data is an average over the two identical configurations \((3, 2)\) and \((2, 3)\).

**Supplementary noise data at \( T_0 = 40 \, \text{mK} \).** We here demonstrate the robustness of our result with respect to the base temperature \( T_0 \). Supplementary Fig. S9 displays the data obtained at filling factor \( \nu = 3 \) for a higher temperature \( T_0 = 40 \, \text{mK} \). The range of injected power was adjusted to fulfill the criteria of dominant electronic heat flow in the reference configuration \( n = 4 \) (\( nJ_Q^e \gtrsim J_Q^{e-\text{ph}} \) at \( n = 4 \)). We used the same analysis as the one presented in the main text, the fit of the reference \( n = 4 \) signal (continuous line in supplementary Fig. S9A) yielding \( \Sigma \Omega = 5.5 \, \text{nW/K}^5 \) for the electron-phonon coupling and \( \alpha_4 = 3.8 \) for the electronic heat flow. Our extraction of the electronic heat currents (supplementary Fig. S9B) is in reasonable agreement with the expected value \((n-4) \times \frac{\pi^2 k_B^2}{6\hbar} \) displayed as continuous lines for \( n \) running from two to four. The ‘model-free’ analysis detailed in the article yields for \( \alpha'_{n-4} \), the value \(-2.2 \frac{\pi^2 k_B^2}{6\hbar} \) at \( n = 2 \) and \(-1.2 \frac{\pi^2 k_B^2}{6\hbar} \) at \( n = 3 \).
Figure S1: Noise measurement setup. (A) Simplified schematic description: the sample is connected in parallel to a $R_p/L/C$ resonator; the signal is then amplified by an home-made cryogenic preamplifier followed by a room temperature amplifier. The listed $R_p, L, C$ values of the resonator correspond to the measurement chain 2, behind QPC$_2$, which was used for the data shown in the article. (B) Practical implementation: The resonator inductance $L$ is made of a superconducting coil thermally anchored to the mixing chamber. The resonator capacitance $C$ is mostly given by the distributed capacitance along the coaxial cables connecting the sample to the cryogenic preamplifier. The parallel shunt resistor $R_p$ models the dissipation mostly due to the $\sim 100 \, \Omega$ resistance of the coaxial wire between sample and superconducting coil. Note the presence of a DC block 4.7 nF capacitance.
Figure S2: Amplification chain calibration. Symbols correspond to the noise measured at the output of the full amplification chain after integration over the measurement bandwidth $\delta f$, for the two investigated filling factors $\nu = 3, 4$. The data is plotted versus the temperature readings $T_{\text{RuO}_2}$ of a RuO$_2$ sensor thermalized to the mixing chamber. Continuous lines are linear fits of the data at $T_{\text{RuO}_2} \geq 50$ mK. Essentially, the gain and the system noise offset can be inferred from, respectively, the slope and the intersect at $T = 0$. Note that the larger offset at filling factor $\nu = 4$ mostly results from the choice of a broader integration bandwidth $\delta f$. 

\[ S_V (10^{-3} V^2) \]

\[ T_{\text{RuO}_2} (\text{mK}) \]
Figure S3: Characterization of the resonant circuit. (A) Equilibrium voltage noise spectrum measured at the output of the amplification chain 2, for filling factor $\nu = 3$. Each trace corresponds to a different temperature $T_{\text{RuO}_2}$: from bottom to top, 50, 150, 250, 350 and 520 mK. (B) Linearity of the noise as a function of temperature. The symbols represent the extracted values of the above noise spectra at five different frequencies, indicated by the dashed lines in the previous panel. The continuous lines are linear fits of these data. (C) Slope of the temperature dependence of the noise spectrum, at filling factors $\nu = 2, 3, 4$. The symbols are the slopes obtained by applying the linear fit procedure shown in panel B to every frequency of the spectrum. The continuous lines are fits using supplementary Eq. S2 and the model shown in supplementary Fig. S1A. (D) Voltage noise added by the amplification chain, referenced to the input. The symbols are extracted from the $T = 0$ intercept of the linear fits, at filling factors $\nu = 2, 3, 4$. 
Figure S4: Comparison of the noise measurements from both amplification chains. Measured noise spectral density at $\nu = 3$ and $T_0 = 24$ mK versus injected DC current. The symbols correspond (from bottom to top) to $n = 0, 2, 3$ and 4 open channels. Open symbols: measurement with amplification chain 1. Filled symbols: measurement with amplification chain 2.
Figure S5: Typical conductance traces measured at $\nu = 4$ for QPC$_1$ (left panel) and QPC$_2$ (right panel), plotted as a function of the corresponding applied gate voltage.
Figure S6: Comparison of measured noise for symmetric sample configurations. (A) Measured noise spectral density at \( \nu = 3 \) and \( T_0 = 24 \) mK versus injected DC current, for the \((n_1 = 1, n_2 = 2)\) (filled symbols) and \((n_1 = 2, n_2 = 1)\) (open symbols) configurations. (B) Measured noise spectral density at \( \nu = 4 \) and \( T_0 = 22.5 \) mK versus injected DC current, for the \((n_1 = 1, n_2 = 2)\) (filled circles), \((n_1 = 2, n_2 = 1)\) (open circles), \((n_1 = 2, n_2 = 3)\) (filled diamonds) and \((n_1 = 3, n_2 = 2)\) (open diamonds) configurations.
Figure S7: Noise measurements on the full span of injected power $J_Q$, at $\nu = 3$ and $T_0 = 24$ mK. The electronic temperature $T_\Omega$ in the micron-sized ohmic contact is plotted as a function of the injected power. The different set of symbols correspond (from top to bottom) to $n = 2$, 3 and 4 open channels. The size of the symbols indicates the uncertainty. The continuous line is a fit of the reference data for $n = 4$ open channels including the heat transfer toward phonons. Note that the fit was performed in the low injected power range $J_Q \lesssim 20$ fW where the electronic heat flow is dominant, and evaluated with the same parameter up to the maximum injected power $J_Q \simeq 90$ fW. The dashed lines correspond to the expected contribution of the electronic heat flow $nJ_Q$ in absence of electron-phonon cooling. Inset: zoom on the data for the restricted span of $J_Q$ shown in the main text, emphasizing the fact that on this span, electronic heat flow and electron-phonon cooling have similar contributions.
Figure S8: Noise measurements at $\nu = 4$ and $T_0 = 22.5$ mK. Variation of the electronic temperature in the micron-sized ohmic contact $T_{\Omega}$ as a function of the injected power, at $\nu = 4$ and $T_0 = 22.5$ mK. The symbols correspond (from top to bottom) to $n = 2, 3, 4, 5$ and 6 open channels. The size of the symbols indicates the uncertainty. The continuous line is a fit of the reference data for $n = 4$ open channels, including the heat transfer toward phonons. Inset: raw noise data versus injected DC current.
**Figure S9:** Noise measurements and corresponding electronic heat current at $\nu = 3$ and $T_0 = 40$ mK. (A) Variation of the electronic temperature in the micron-sized ohmic contact $T_{\Omega}$ as a function of the injected power. The symbols correspond (from top to bottom) to $n = 2$, 3, and 4 open channels. The continuous line is a fit of the reference data for $n = 4$ open channels. Inset: raw noise data versus injected DC current. (B) Symbols display the heat current across $|n - 4|$ electronic channels, with a positive (negative) sign for $n > 4$ ($n < 4$), versus squared temperature, at $T_0 = 40$ mK and filling factor $\nu = 3$, for (from bottom to top) $n = 2$, 3 and 4 open channels. The continuous lines are the theoretical predictions for the quantum limited heat flow.
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**Table S1:** Extracted values $\alpha'_{n-4}$ (model-free approach) and $\alpha_n$ (model-dependent approach).
References and Notes


25. Materials and methods are available as supplementary material on *Science* Online.


