1. Materials and sample preparation

High-quality single crystals of Sr$_2$RuO$_4$ used in this work were grown by the floating-zone technique and characterized in detail by standard techniques (SI). We found in the course of this work that crystals with an optimal superconducting transition temperature ($T_c$) around 1.5 K typically have more Ru inclusions than those with slightly lower $T_c$. However, those with too low a $T_c$ (around 1 K), most likely due to Ru deficiency, tend to cleave easily. Therefore we chose crystals with a $T_c$ around 1.4 K.

To prepare a Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ Geshkenbein, Larkin, and Barone (GLB) SQUID, a crystal was cut perpendicular to a cleaved $ab$ plane and polished using non-magnetic abrasives with a varying average size down to as small as 0.1 µm to form two parallel surfaces perpendicular to the $ab$ plane (the RuO$_2$ plane). The polished surfaces were inspected carefully under high power (500×) optical microscope to see if Ru inclusions were present on the surface. We found that in this way essentially all Ru inclusions on the surface could be identified. An insulating layer (1500 Å SiO) was deposited by thermal evaporation on the $ab$ plane (Samples A and B, Fig. 1A), or on a third polished face, perpendicular both to the junction faces and to the $ab$ plane (Sample C). Polishing a third face increases the chance that the crystal will cleave unintentionally but results in a smaller effective SQUID area when it is successful, as described below. The conventional, $s$-wave superconductor, Au$_{0.5}$In$_{0.5}$, was evaporated onto carefully masked, Ru-inclusion-free areas of the polished surfaces. The two Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ junctions
of the SQUID were prepared simultaneously by rotating the crystal during the evaporation of Au and In. All thermal evaporations were done in a high-vacuum evaporator with a residual pressure in the range of (1-5)×10⁻⁶ Torr and with high-purity Au and In (99.9999% pure). Two measurement leads were attached with silver epoxy (cured on a hot plate) to both the Au₀.₅In₀.₅ and to the Sr₂RuO₄ to enable four-wire measurements.

The same-side (SS) SQUID and corner-junction samples for control experiments were prepared using a similar procedure. For corner junctions, we aligned the crystal for polishing so that the two mutually perpendicular planes were as parallel to the c axis of Sr₂RuO₄ as possible. The corner was reasonably sharp, rounded at the micron scale.

In preparing all SQUIDs and corner junctions, we did not attempt the demanding job of aligning the polished surfaces with the a and b axes. It is unlikely that the junction surfaces happen to be the ac or bc planes.

Results on the material and superconducting properties of Au₁₋ₓInₓ have been summarized previously (S2). In the bulk form, depending on the In composition x, the values of Tc of Au₁₋ₓInₓ were reported to range from 0 K (for pure Au) to 0.4-0.6 K for intermetallic compound AuIn (S3). Au₁₋ₓInₓ was found to wet Sr₂RuO₄ (and other substrates) extremely well and to possess a long superconducting coherence length (S4), both of which are desirable for the preparation of a high-quality Josephson junction on Sr₂RuO₄. The atomic composition x = 0.5 was chosen because its bulk Tc (0.4-0.6 K) was the highest in this series of alloys. We followed the procedures established previously (S4) and deposited alternating layers of Au and In onto the polished surfaces with appropriate thicknesses to ensure equal atomic concentrations of the two metals. The thickness of each layer was kept below 100 Å. Previous studies (S4) showed that at such layer thickness, Au and In interdiffuse fully at room temperature.
Figure S1. Resistance vs. temperature for a Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ GLB SQUID and its Au$_{0.5}$In$_{0.5}$ film electrode in zero magnetic field.

The $T_c$ of a Josephson junction of two dissimilar materials can in principle be different from the lower $T_c$ of the two materials. For our Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ SQUIDs, the area of Au$_{0.5}$In$_{0.5}$ film is usually small, allowing the attachment of only two measurements leads. Therefore the film and the junction $T_c$s were not determined at the same time for most samples. However, in the case in which the two were both measured, they were found to be essentially the same (Fig. S1).

2. The 3 K phase in Sr$_2$RuO$_4$

The task of ensuring the complete absence of the 3 K phase of Sr$_2$RuO$_4$—the pure Ru inclusions embedded in the single-crystal primary phase with a $T_c$ higher than that of the bulk Sr$_2$RuO$_4$, which can be as high as 3 K—at the junction interfaces of the SQUID is a demanding one. However, this is important as the presence of Ru inclusions at the junction interface can potentially introduce serious complications. Since bulk Ru has a $T_c$ of 0.5 K, the Ru inclusions
formed in Sr$_2$RuO$_4$ are clearly a superconductor different from bulk Ru. It has been shown theoretically (S5) that the pairing symmetry of the Ru inclusions is the same as that of the bulk Sr$_2$RuO$_4$ when the latter is superconducting. If a Ru inclusion happens to be located at the interface, the order parameter in the Ru is expected to have the same symmetry as in the bulk Sr$_2$RuO$_4$ crystal when the latter is superconducting. Therefore, the presence of Ru inclusions will not affect the qualitative nature of the Josephson coupling between a conventional superconductor and Sr$_2$RuO$_4$ (S6). Previous studies (S7) indicated that above the bulk $T_c$ of Sr$_2$RuO$_4$, the Ru inclusions are also unconventional, as predicted (S5).

![Figure S2](image_url)

**Figure S2.** Close-up plot of sample resistance $R(T)$ for three opposite-side Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ GLB SQUIDs in the normal state. For Samples A and B, the absence of any features between 1.5 and 3 K indicate that no Ru inclusions were present at the interface. For Sample C, the presence of a small inclusion near or at the junction interface is indicated by the change in slope of $R(T)$ at 1.7 K (see text).

The main complication due to the presence of Ru inclusions at the junction interface is that, because of the differences in the electronic band structures and other material properties between Ru metal and Sr$_2$RuO$_4$, the tunnel barriers between Ru and Au$_{0.5}$In$_{0.5}$ and that between
Sr$_2$RuO$_4$ and Au$_{0.5}$In$_{0.5}$, will be different. The presence of Ru inclusions at the junction interface will therefore introduce uncontrolled “shorts” at the junction, making the effective areas of the two junctions in the SQUID different. The asymmetry introduced this way will in turn lead to a circulating current, and therefore induced flux.

Another important, but subtler issue is related to the relative phase between $c_{21}$ and $s_{21}$ in Eq. 1. The idea of using an opposite-side GLB SQUID to test odd-parity superconductivity relies on a special “symmetry” relation in free energies of the two oppositely-facing junctions (S8). Therefore it will help if the relative phase between $c_{21}$ and $s_{21}$ is fixed. It has been speculated (S9) that the difference in the electronic properties between an oxide (Sr$_2$RuO$_4$) and a metal alloy (Au$_{0.5}$In$_{0.5}$) will lead to a definitive relative phase between $c_{21}$ and $s_{21}$. While more theoretical work is needed to clarify this issue, it is possible that the presence of Ru inclusions at the junction interface could invalidate such a constraint, leading to more complicated behavior.

For Samples A and B of the GLB Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ SQUIDs, no Ru inclusions are present at the junction interface based on careful examination of the optical images of the crystal surface and the temperature dependence of the sample resistance, $R(T)$. For Sample C, however, an indication of the presence of Ru inclusions with $T_c$ about 1.7 K, was found in $R(T)$ (Fig. S2), even though no Ru inclusion was visible under optical microscope at 500× magnification. The suppressed $T_c$ (from the usual value of about 2.5–3 K) and the extremely small drop in $R(T)$ at 1.7 K indicate that the size of Ru inclusion(s) is extremely small, or perhaps more likely, the Ru inclusion was fully embedded in Sr$_2$RuO$_4$ crystal (therefore invisible under the optical microscope) but close to the junction surface. In any case, fortunately, the presence of Ru inclusion in this particular sample appears to have led to no complications.
3. Magnetic shielding and measurements

The Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ SQUID samples were shielded magnetically to minimize the background field. First, multiple layers of mu-metal tape (Metglas 2714A, Allied Signal Advanced Materials) were wrapped on a capped Pb can (15 cm in length and 2.6 cm in diameter) to form a capped mu-metal shield. A slightly shorter, also capped Al can of an inner diameter of 2.2 cm was fitted inside the Pb can. A homemade coil of superconducting wire, 12.6 cm long, fitted inside the Al can, generated the magnetic field. The field coil was kept slightly away from the cap of the Al can (>1 cm) to avoid overly compressing the field lines at this end of the coil. The samples sit at the middle of the field coil, about 7.6 cm from the open end of the field-shielding can. The background field was estimated to be substantially less than 1 mG based on the geometry and the design of the shielding assembly, and the measurements carried out in a similar shielding assembly made previously ($S10$, $S11$).

The samples were cooled in a dilution refrigerator with a base temperature of 20 mK. Low-pass, radio-frequency filters were installed on top of the cryostat to filter all electrical leads entering the measurement space. Measurements of sample resistance, $I$-$V$ curves, and critical currents were taken using a direct current source and a nanovoltmeter. The field and the temperature dependences of the critical current were measured by bisecting an initial current (> $I_c$) repeatedly until the critical current was found (a small, fixed voltage (40-50 nV) was detected). Given that the normal-state resistance of the SQUID is in the range of a few ohms (Fig. 1C), this corresponds to a detection limit in $I_c$ in the range of a few tens of nA. We compared the values of $I_c$ obtained this way and that obtained under identical conditions by measuring $I$-$V$ curves directly and found that they were essentially the same.
4. Inductance of \( \text{Au}_{0.5}\text{In}_{0.5}\text{–Sr}_2\text{RuO}_4 \) GLB SQUIDs

As a self-consistence check, we estimate the inductance of our \( \text{Au}_{0.5}\text{In}_{0.5}\text{–Sr}_2\text{RuO}_4 \) SQUIDs using two different methods. From \( T = 0.21 \) to 0.42 K, \( I_c(H, T) \) shifted by \( \delta H = 1.1 \text{ mG} \) (corresponding to \( \delta \Phi = 0.13 \Phi_0 \)) and \( \delta I_c = 3.0 \text{ µA} \) for sample A (Fig. 4A). The shifts from 0.2 K to 0.5 K are \( \delta H = 1.5 \text{ mG} \) (\( \delta \Phi = 0.13 \Phi_0 \)), \( \delta I_c = 8.8 \text{ µA} \) for sample B and \( \delta H = 18 \text{ mG} \) (\( \delta \Phi = 0.18 \Phi_0 \)), \( \delta I_c = 2.5 \text{ µA} \) for Sample C (Fig. S3). If we approximate the change in \( I_c \) roughly by \( \delta I_c/2 \), we can estimate \( L \) using \( \delta \Phi = L \delta I_c/2 \) (S12). The values of the total inductance \( L \) so obtained are \( L = 0.18 \text{ nH} \) for Sample A, \( L = 0.041 \text{ nH} \) for Sample B, and \( L = 0.30 \text{ nH} \) for Sample C.

The values of \( L \) can also be estimated from sample dimensions although this is not straightforward given the shape of our SQUID loops (Fig. 1A). Using a formula (S13) for long and narrow inductor loops, \( L = K_0 \mu_0 \ell \), where \( \ell \) is the length of loop (= \( h/2 \) for our SQUIDs), \( \mu_0 \) is permeability of free space (\( = 4\pi \times 10^{-7} \)), and \( K_0 \) is a number of the order of unity, which overestimates \( L \), we obtain a \( L \) value around 0.32 nH for Samples A and B, and 0.19 nH for Sample C if we take \( K_0 = 1 \). However, if we use a different formula for short and wide inductor loops (S14), \( L = \mu_0 A/d \), where \( A \) is the effective area of the SQUID loop, \( d \) is the depth of the SQUID (Fig. 1A), which underestimates \( L \), we have \( L \) around 8.1 pH for Sample A, 18.2 pH for Sample B, and 0.6 pH for Sample C. Given the experimental uncertainties involved, the consistency of these estimated values of \( L \) should be considered satisfactory.

5. Additional SQUID samples

Nearly 30 opposite-side \( \text{Au}_{0.5}\text{In}_{0.5}\text{–Sr}_2\text{RuO}_4 \) GLB SQUIDs were prepared over the course of this experiment. Four GLB SQUIDs were found to exhibit a finite supercurrent at earlier stages of this work, along with preliminary evidence for quantum interference. At the time,
imperfections in measurement technique, the magnet coil (inside the magnetic shielding), and the sample orientation prevented us from characterizing the interference pattern. As a result, we were unable to draw a definite conclusion about the location of the minimum in $I_c(H)$. Four samples prepared more recently were again found to possess a finite supercurrent, and an interference pattern in $I_c(H)$. Among them, Sample A, B, and C were quite symmetric. These samples were prepared in two orientations: Samples A and B are of the first type, as shown in Fig. 1A. Sample C is of the second type in which the $c$ axis of the Sr$_2$RuO$_4$ crystal is perpendicular to, rather than parallel with, the plane of the SQUID loop. This second type of SQUID is associated with the smaller penetration depth $\lambda_2$ (about 20 times smaller than $\lambda_1$) on all three sides. This makes the effective area of the SQUID smaller, and the period larger, than the first type. This is a better situation to offset the effect of the background magnetic field on the total flux threading the SQUID. In all three samples, the minimum in the $I_c(H)$ interference pattern was found to shift towards $H = 0$ as $T$ was increased to $T_c$, as shown in Fig. 4A and Fig. S3. A fourth sample (data not shown) was particularly susceptible to trapped flux, which was evidenced by the frequent jumps in $I_c(H)$ during the field sweeps. This was probably due to the presence of imperfections in the junctions rather than intrinsic properties of the superconductors. The trapped flux prevented a definitive determination of the location of the minimum in the interference pattern.
Figure S3. (a) $I_c(H)$ for GLB Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ SQUID Sample B taken close to $T_c$ of the SQUID. From top to bottom, $T = 0.200, 0.250, 0.275, 0.300, 0.350, 0.370, 0.400, 0.425, 0.450, 0.475, and 0.500$ K. The period for the oscillation is $(15.7 \pm 0.4)$ mG. (b) $I_c(H)$ for GLB Sample C at $T = 0.200, 0.300, 0.350, 0.400, 0.420, 0.440, 0.460, 0.470, 0.480, 0.490,$ and $0.520$ K. The period is $(98 \pm 2)$ mG.

After the extensive initial measurements, Sample A was cooled down again after a thermal cycle to room temperature (AuIn–SRO A-2). While the relevant features in $I_c(H)$ were unchanged, the thermal cycling did lead to a reduction of the critical current and a different asymmetry, as indicated by the different direction of the shift with increasing temperature. Therefore, we effectively had a different GLB SQUID sample. During the second cooling, improved measurement techniques (described below) enabled us to succeed with measurements closer to $T_c$, where $I_c$ was closer to zero and the induced flux due to the asymmetry of the SQUID was further minimized, without introducing trapped flux. The most notable feature of this set of data is that the minimum of $I_c(H)$ at the highest temperature was found to be very close to $H = 0$ (Fig. S4). This confirms the robustness of our results that support the odd-parity superconductivity picture for Sr$_2$RuO$_4$. 
Figure S4. $I_c(H)$ for GLB $\text{Au}_{0.5}\text{In}_{0.5}-\text{Sr}_2\text{RuO}_4$ SQUID Sample A-2 taken close to $T_c$ of the SQUID after a thermal cycle to 300 K. From top to bottom, $T = 0.320, 0.360, 0.400, 0.405, 0.407, 0.410, 0.420, 0.430$, and $0.450$ K. Except for the curve at $T = 0.450$ K, the curves are shifted upwards (by 0.1, 0.3, 0.5, 0.7, 1.1, 1.4, 1.6, and 2.0 $\mu$A, from bottom to top) for clarity.

The overall picture that has emerged from the extensive measurements of these four opposite-side GLB SQUID samples (five different sets of data, including two separate coolings of Sample A), does not seem to support an earlier theoretical suggestion ($S15$) that, roughly speaking, only half of the GLB SQUID samples will lead to the results predicted originally by GLB. We believe that the suggested bimodal result may indeed emerge if the $d$-vector in $\text{Sr}_2\text{RuO}_4$ were in-plane $(ab$ plane direction) ($S16$) rather than the $c$ axis as shown by the previous ($S17$) and the present experiments.

A similar number of same-side (SS) SQUID control samples was prepared. The success rate for these control samples is similar to that of the opposite-side GLB SQUID samples. Among all the SS samples we measured, three samples were found to possess a finite supercurrent
and an interference pattern in $I_c(H)$. The period of the $I_c(H)$ oscillations for all these SS samples is consistent with the sample dimensions. In addition to SS SQUID Sample A shown in Fig. 4B of the main text, results obtained in SS SQUID Samples B and C are shown in Fig. S5. For SS SQUID Sample B, there appears to be a double-period oscillation, which may be due to the presence of inhomogeneities in one of the junctions. However, the overall pattern of the $I_c(H)$ is quite symmetric. For Sample SS Sample B, the overall pattern of the $I_c(H)$ is not as symmetric as for the other two samples, most likely due to the asymmetry of the SQUID. However, the $I_c(H)$ pattern was found to shift systematically as $T$ was raised, and no sign of flux trapping was found during the measurements. Results from all three SS SQUID control samples prepared on the same surface of the crystal perpendicular to the $ab$ plane clearly show that the $I_c(\Phi)$ interference pattern has a maximum rather than minimum at $\Phi = 0$, in strong contrast with the opposite-side GLB samples.

**Figure S5.** (a) $I_c(H)$ for SS Sample B. From top to bottom, $T = 0.030, 0.100, 0.015, 0.200, 0.225, 0.250, 0.275$, and $0.300$ K. (b) $I_c(H)$ for SS Sample C at $T = 0.200, 0.250, 0.300, 0.350, 0.400$, and $0.460$ K.
A fourth same-side SQUID (SS Sample D) did not show a measurable $I_c$ at the lowest temperature (30mK), but nevertheless exhibited quantum interference in resistance as a function of applied field. Given that similar samples led to useful information in the high-$T_c$ work (SI8), we pursued systematic $R(H)$ measurements on this sample as well. Figure S6a shows the results of $R(H)$ taken at several currents. While a double-period oscillation was found, most likely due to junction inhomogeneities again, the $R(H)$ interference pattern of this SS SQUID clearly has a minimum at $H = 0$. As a comparison, Fig. S6b shows $R(H)$ for an opposite-side GLB Au$_{0.5}$In$_{0.5}$–Sr$_2$RuO$_4$ SQUID (Sample C) for several currents larger than $I_c$, showing that the $R(H)$ interference pattern of this GLB SQUID has a maximum at $H = 0$. These results are consistent with the $I_c(H)$ measurements, and provide additional support to odd-parity superconductivity in Sr$_2$RuO$_4$.

**Figure S6.** Quantum interference in $R(H)$ for (a) SS Sample D measured using currents of 0.05, 0.50, 1.0, 1.1, and 1.5 µA at $T = 0.03$ K, and (b) GLB Sample C measured using currents of 1.3, 1.5, 1.7, and 1.9 µA at $T = 0.5$ K.
6. Extrema in $I_c$ vs. $H$ as a function of temperature

As shown in Fig. 4 of the main text, and in Figs. S3 to S5, the interference patterns of $I_c(H, T)$ shift as the temperature increases to $T_c$. Fig. S7a shows the temperature dependence of the applied field $H$ at which $I_c$ shows a minimum near zero field for the opposite-side GLB SQUID samples. The bumps around 0.3 K in GLB Samples A and B (Fig. S7a) may indicate that the temperature dependence of $I_a$ and $I_b$ in the SQUID may have slightly different temperature dependences, resulting in a slight non-monotonic behavior in $I_c(T)$ for GLB Sample B (but not for Sample A, see Fig. 2B). In any case, it is clear that $H_m(I_c=\text{min})$ approaches zero as $T$ approaches $T_c$. For the SS Au$_{0.3}$In$_{0.5}$-Sr$_2$RuO$_4$ SQUID control samples, a maximum in $I_c(H, T)$ is expected at $H = 0$ as $T \to T_c$ (when the circulating current is minimized), and is observed experimentally as shown in Fig. S8a.

A different way of summarizing the same sets of data is presented in Figs. S7b and S8b. The value of the magnetic field at which a minimum or maximum in $I_c$ occurred nearest zero-field in the interference pattern ($H_m$) is plotted as a function of $I_c^{\text{min}}$ (or $I_c^{\text{max}}$) for each type of sample. It is clear from the plots that $I_c(\Phi=0)$ corresponds to a minimum for the GLB samples and a maximum for the SS control samples as $I_c \to 0$.

Note that at the highest temperatures shown, the actual value of $I_c$ is 0.2 $\mu$A, 2 $\mu$A, and 1 $\mu$A for GLB Samples A, B, and C respectively. In principle, the measurement could continue until $I_c h/2e = k_B T$ (S19), or about $I_c^{\text{min}}=0.02$ $\mu$A for $T = 0.5$ K, but we found that in this case, the maximum applied field had to be kept very small to prevent the trapping of flux in the sample. As a result, as $T$ gets too close to $T_c$, the measurement of the interference pattern over a sufficiently large range of field, needed to determine if the pattern is symmetric, becomes difficult. However, the measurement techniques continued to be improved during the course of this work. After the initial measurements, GLB SQUID Sample A was cooled down again after
a thermal cycle to room temperature (AuIn–SRO A-2). This second series of measurements revealed that the symmetry of the pattern in $I_c(H, T)$ could be verified at low temperature. Then, the position of the minimum could be traced continuously as the temperature was raised while using only a very small applied field. In this way, the data was taken until $I_c$ was even closer to zero than in the first set of measurements (Figs. 4A and S4).

![Graph](attachment:image.png)

**Figure S7.** Values of the magnetic field at which a minimum in $I_c$ was found near zero field in the interference pattern as a function of temperature (a) and as a function of the corresponding minimum critical current (b). $H(I_c=\text{min})$ approaches 0 as $T$ approaches $T_c$. The corresponding values for the flux at the highest temperature measured are $0.16\Phi_0$, $0.10\Phi_0$, and $0.07\Phi_0$, for samples A, B, and C, respectively. A second cooling of Sample A following warming up to 300 K is also shown (GLB Sample A-2).
**Figure S8.** Values of the magnetic field at which a maximum in $I_c$ was found near zero field in the interference pattern as a function of temperature (a) and the corresponding minimum critical current (b). $H(I_c=\text{max})$ approaches 0 as $T$ approaches $T_c$.

### 7. Corner Junctions

As an independent verification of the results obtained in the GLB and SS SQUID samples, we also prepared and measured corner junctions of the type used in the high-$T_c$ research ($S18$). As described above, the corner junction configuration is two junction planes perpendicular to one another and parallel to the $c$ axis. The $s$-wave superconductor, Au$_{0.4}$In$_{0.5}$, was deposited on both planes continuously across the corner to form a single junction. At $H=0$, the interference pattern for such a corner junction is known to exhibit a maximum for an $s$- and a minimum for a $d$-wave superconductor (independent of asymmetry in the sample) based on previous phase-sensitive work on high-$T_c$ superconductors ($S18$). For odd-parity pairing, in the two-dimensional representation for the tetragonal crystal (applicable to Sr$_2$RuO$_4$) with $d$ along the $c$ axis, the $\Gamma_5^-$ state ($S20$), the interference pattern is expected to show neither a minimum nor a maximum at $H=0$, as observed experimentally (Fig. S9). This results not only excludes $s$- and $d$-wave symmetries but also provides support for a pairing symmetry of $\Gamma_5^-$ in Sr$_2$RuO$_4$. Thus,
the experimental results obtained from this corner junction support not only the odd-parity superconductivity, but also the presence of a complex order parameter in Sr$_2$RuO$_4$. So far, we have found a finite supercurrent and interference pattern only in one corner junction among the several samples we prepared. Further work on this important issue of the complex nature of the superconducting order parameter in Sr$_2$RuO$_4$ is needed, and is currently underway.

![Graph](image)

**Figure S9.** Quantum interference in $I_c(H)$ for a corner junction on two adjoining $ac$ faces of Sr$_2$RuO$_4$ at $T = 0.20, 0.30, 0.35,$ and $0.40$ K. Little shift is visible with temperature because the induced flux is small compared to the oscillation period. The period is large because only a single junction, even though it encompasses a corner, is involved.
References and Notes


