Supporting Online Material for

Toroidal Dipolar Response in a Metamaterial

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Radiating power of multipoles. The relative strength of the induced multipole moments contributing to the far-field response of the toroidal metamaterial was evaluated based on the general expression for the total power of multipole radiation derived in (S1) to the 5th order of (1/c). In the case of harmonic excitation \( \sim \exp(i\omega t) \) it has the form

\[
I = \frac{2\omega^4}{3c^3} |\mathbf{P}|^2 + \frac{2\omega^4}{3c^3} |\mathbf{M}|^2 + \frac{4\omega^4}{3c^4} (\mathbf{P} \cdot \mathbf{T}) + \frac{2\omega^6}{3c^5} |\mathbf{T}|^2 + \frac{\omega^6}{5c^5} Q_{apf} Q_{apf} + \frac{\omega^6}{20c^5} M_{apf} M_{apf} + \
+ \frac{2\omega^6}{15c^5} (\mathbf{M} \cdot \langle \mathbf{R}_M^2 \rangle) + \mathcal{O} \left( \frac{1}{c^5} \right)
\]

(S1)

For the purpose of our analysis the multipole contributions were represented in a Cartesian co-ordinate system. The first two terms correspond to the conventional electric (charge) and magnetic dipole scattering. The third term accounts for the interference between the electric and toroidal dipoles, which is generally non-zero because both types of dipoles have the same angular momentum and parity properties. The fourth term corresponds to the toroidal dipole scattering. The fifth and sixth terms come from electric and magnetic quadrupoles. The last term is a further correction resulting from the interference between the magnetic dipole and the first-order mean-square radius of the magnetic dipole distribution \( \langle \mathbf{R}_M^2 \rangle \) (S1, S2). Its radiating power at the frequency of toroidal resonance is several orders of magnitude lower than that of the induced toroidal dipole moment.

For the purpose of our calculations we used the following representations of the multipoles and mean-square radius of the magnetic dipole distribution (S1):

- electric dipole moment: \( \mathbf{P} = \frac{1}{i\omega} \int j d^3 r \), \hspace{1cm} (S2.1)
- magnetic dipole moment: \( \mathbf{M} = \frac{1}{2c} \int (\mathbf{r} \times j) d^3 r \), \hspace{1cm} (S2.2)
- toroidal dipole moment: \( \mathbf{T} = \frac{1}{10c} \int ((\mathbf{r} \cdot j)\mathbf{r} - 2r^2 j) d^3 r \), \hspace{1cm} (S2.3)
electric quadrupole moment:  
\[ Q_{\alpha\beta} = \frac{1}{i\omega} \int \left[ r_{a} j_{\beta} + r_{\beta} j_{a} - \frac{2}{3} (\mathbf{r} \cdot \mathbf{j}) \right] d^{3}r, \]  
\hspace{1cm} (S2.4)

magnetic quadrupole moment:  
\[ M_{\alpha\beta} = \frac{1}{3c} \int \left[ (\mathbf{r} \times \mathbf{j})_{\alpha} r_{\beta} + (\mathbf{r} \times \mathbf{j})_{\beta} r_{\alpha} \right] d^{3}r, \]  
\hspace{1cm} (S2.5)

mean-square radius of magnetic dipole distribution:  
\[ \langle R_{M}^{2} \rangle = \frac{1}{2c} \int (\mathbf{r} \times \mathbf{j}) r^{2} d^{3}r, \]  
\hspace{1cm} (S2.6)

where \( \mathbf{j} \) is the current density and \( c \) is the speed of light.

Here charge density \( \rho \), which usually appears in the definition of electric dipole and quadrupole, has been replaced with current density \( \mathbf{j} \) using the continuity equation

\[ i\omega \rho(\mathbf{r}) + \text{div} \mathbf{j}(\mathbf{r}) = 0 \]  
\hspace{1cm} (S3)

**Numerical modeling of the metamaterial response.** Electromagnetic response of the toroidal metamaterial, including spatial distribution of the induced currents required in formulas S2.1-S2.6, was modeled using 3D Maxwell’s equations solver of the Comsol Multiphysics simulation package. Metamaterial array was represented by its metamolecule placed in a rectangular unit cell (see Fig. 2A in the main text) with periodic boundary conditions imposed along \( z \)- and \( z' \)-axes. The following values were used for the dimensions of the unit cell and the metamolecule: \( d = 8.00 \) mm, \( s = 7.50 \) mm, \( a = 1.80 \) mm, \( r = 2.44 \) mm, \( h = 1.50 \) mm, \( w = 0.15 \) mm and \( g = 0.15 \) mm. The wires of the rectangular metallic loops were assumed to be perfect electric conductors. Permittivity of the dielectric slab was set to \( \varepsilon = 3.0 - i 0.0039 \) (corresponds to low-loss microwave laminate Roger 3003).

**Sample fabrication and experimental measurements.** The metamaterial slab was formed by a 22 \( \times \) 22 array of toroidal metamolecules, which had an overall size of about 6.0 \( \times \) 6.0 \( \times \) 1.5 mm\(^3\). The rows of the array were manufactured from strips of a copper-coated low-loss laminate Roger 3003 using the high-resolution printed circuits board technology. At microwave frequencies this laminate has the dielectric constant of 3.00 with a very small dissipation factor of 0.0013 (S3). The copper tracks on both sides of the strips forming the top and bottom sections of the loops had the cross-section of 0.035 \( \times \) 0.150 mm\(^2\) and were coated with 0.002 mm thick layer of gold. Due to limitations of the printed circuit board technology, the vertical segments of the wire loops embedded into the dielectric of the strips were fabricated as electroplated through channels with the diameter of 0.2 mm. The metamaterial array was assembled by vertically stacking the strips at the regular interval of 7.5 mm, such that the axis of all toroidal metamolecules laid in the plane of the array, i.e. parallel to the front and back sides of the resulting metamaterial slab (Fig. S1).

The transmission and reflection spectra of the metamaterial slab were measured at normal incidence in a microwave anechoic chamber using a vector network analyzer Agilent E8364B and linearly polarized horn antennas Schwarzbeck M. E. BBHA 9120D equipped with dielectric lenses. The polarization of the antennas was set orthogonal to the dielectric strips with toroidal metamolecules (i.e. parallel to the axes of the metamolecules).
Fig. S1. Close-up photograph of assembled metamaterial slab

References


S3 http://www.rogerscorp.com