Comment on "The Geometry of Musical Chords"

Dave Headlam* and Matthew Brown

Tymoczko (Reports, 7 July 2006, p. 72) proposed that the familiar sonorities of Western tonal music cluster around the center of a multidimensional orbifold. However, this is not true for all tonal progressions. When prototypical three-voice cadential progressions by Bach converge on the tonic, the chords migrate from the center to the edge of the orbifold.

Tymoczko recently reported that common sonorities in Western tonal music cluster around the center of a multidimensional orbifold (*1*). The Report specifically states, "Chords that divide the octave evenly lie at the center of the orbifold and are surrounded by the familiar sonorities of Western tonality." Although the orbifold can model stepwise voice leading among complete (and hence near maximally even) triads and seventh chords, and many tonal progressions contain such sonorities, tonal progressions are also characterized by convergence onto octaves and unisons. Consider, for example, the progression shown in Fig. 1. Written in three voices, this progression is taken from Johann Joseph Fux's *Gradus ad parnassum* (*2, 3*), one of the most famous theory treatises. Two interesting properties are worth noting. First, the constituent voices all converge on the tonic at the end (the cadence). The upper voices approach the tonic by step, whereas the bottom voice leaps. Second, the progression avoids parallel perfect octaves and fifths between successive voices (*4*). Despite Tymoczko's claim, however, this progression does not cluster around the center of the orbifold. On the contrary, the chords diffuse to the orbifold's outer edges. They do so because each voice ends on the tonic note; as figure 2 in (*1*) clearly shows, unisons and octaves lie at the edges of the orbifold.

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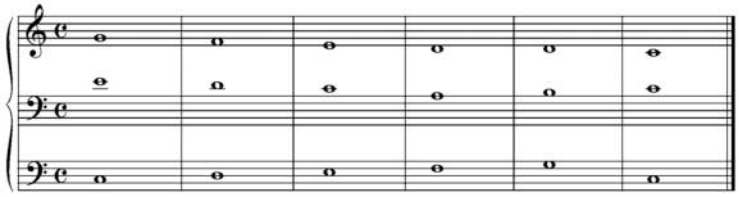

Fig. 1. A three-voice progression taken from (*3*), figure 91, p. 72.

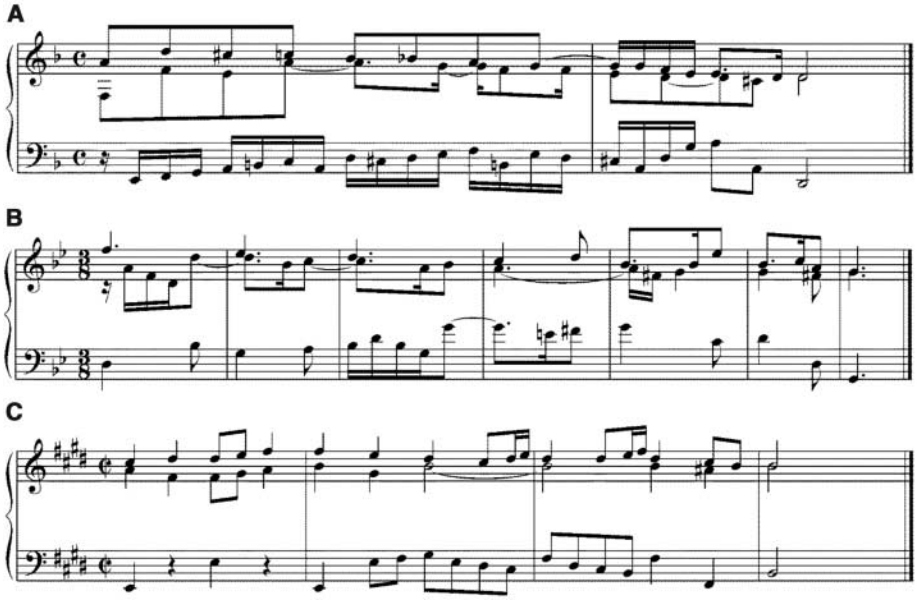

Fig. 2. Examples of three-voice progressions by Bach. (**A**) Fugue a 3, *Well-Tempered Clavier II*, measures 26 and 27. (**B**) *Three-Part Invention No. XI*, measures 66 to 72. (**C**) Gavotte from *French Suite No. VI*, measures 5 to 8.

We chose to illustrate this particular progression because it is tonal and because Fux's treatise has formed the basis of instruction in music theory for almost 300 years. As the historian Alfred Mann has carefully documented, composers such as Haydn, Mozart, Beethoven, and Brahms not only learned from the book but also used it as the basis for their own teaching (*5*). It also served as a point of departure for many later tonal theorists. For example, Heinrich Schenker specifically cited this progression in the second volume of his treatise *Kontrapunkt* (*6–8*). Analogous three-voice progressions are common throughout the standard repertory and include three well-known pieces by Bach: the D minor fugue from his second book of the *Well-Tempered Clavier*, his *Three-Part Invention No. 11 in G minor*, and the Gavotte from his *French Suite No. 6 in E major* (Fig. 2, A, B, and C, respectively). In each case, the progression migrates to the edges of the orbifold because the individual voices all end on the tonic note. This point is important because it highlights that tonal theorists are primarily interested in explaining the sorts of convergent progressions that define tonalities. These progressions are typically found at points of closure (cadences) and play a crucial role in articulating a work's formal structure. This is certainly the case with each of Bach's progressions (Fig. 2). Music psychologists Rosner and Narmour have confirmed the connections between convergence and tonal closure experimentally (*9*). Tymoczko's report does not appear to address this crucial feature of music from the Western tradition.

References and Notes

1. D. Tymoczko, *Science* **313**, 72 (2006).
2. J. J. Fux, *Gradus ad parnassum* (Van Ghelen, Vienna, 1725).
3. J. J. Fux, *Gradus ad parnassum*, A. Mann, Transl. (Norton, New York, 1971).
4. Although Tymoczko alludes to this idea in (*1*), he doesn't formulate it precisely. He says that voices should move "independently (not all in the same direction by the same amount)." Expressed in this way, Tymoczko inadvertently prohibits parallel thirds and sixths, such as those between the top and middle voices in measures 1 to 3 and the middle and bottom voices in measures 4 and 5 of our Fig. 1.
5. A. Mann, *Theory and Practice: The Great Composers as Teachers and Students* (Norton, New York, 1987).
6. H. Schenker, *Kontrapunkt* (Universal, Vienna, 1922).
7. H. Schenker, *Counterpoint II*, J. Rothgeb, J. Thym, Transl., ed. 2 (Musicalia, Ann Arbor, MI, 2001), Example 6, p. 11.
8. The progression is also analogous to one of Schenker's deep-middleground paradigms. [See (*10*).]
9. B. Rosner, E. Narmour, *Music Percept.* **9**, 383 (1992).
10. H. Schenker, *Free Composition*, E. Oster, Transl. (Longman, New York, 1979), fig. 16.2c.

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