Anisotropic Violation of the Wiedemann-Franz Law at a Quantum Critical Point

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A quantum critical point transforms the behavior of electrons so strongly that new phases of matter can emerge. The interactions at play are known to fall outside the scope of the standard model of condensed matter physics, and their extrapolation are shown in detail in (4). For $H = 10 T$, far away from $H_c$, $w(T)$ and $p(T)$ converge as $T \to 0$ for both current directions. However, very close to the QCP, for $H = 5.3 T$, they only converge for in-plane transport. In other words, transport along the $c$ axis violates the WF law, with $w_c$ extrapolating to a distinctly lower value than $w$. In the supporting material (4), we show that extrapolations are not needed to conclude in a violation of the WF law, as the difference data, $w_c(T) - p(T)$ versus $T$, shows a rigid $T$-independent shift from field to field. The normalized Lorenz ratio, $L_c = L/T = p/T$, is also seen to approach unity at 10 T but not at 5.3 T.

Our observation of a violation of the WF law at a QCP is characterized by three distinctive perature dependence of properties at the QCP, for example, a specific heat coefficient that never saturates, growing as $C/T \sim \log(1/T)$ (7), and an electrical resistivity that grows linearly with $T$ (8). Quantum criticality also appears to be linked to the emergence of exotic forms of superconductivity (9–11) and nematic (12) electronic states of matter.

To determine whether Landau quasiparticles survive at a QCP, we have measured the transport of heat and charge in CeCoIn$_5$, a heavy-fermion metal with a QCP tuned by magnetic field $H$. In its phase diagram (Fig. 1), the QCP is located on the border of superconductivity and marks the end of a FL regime at $H_c = 5.0 T$, where the electrical resistivity obeys the FL form $\rho = \rho_0 + AT^\alpha$ (13). A power-law fit to the $A$ coefficient yields $A \sim (H - H_c)^{\alpha}$, with $\alpha \approx 4/3$ and $H_c = 5.0 \pm 0.1 T$ (13). At $H_c$, $C/T$ never saturates (14). The same phenomenon is found at the field-tuned QCP of YbRh$_2$Si$_2$ (with $\alpha \approx 1$) (15).

In Fig. 2, we show how the thermal and electrical resistivities in the $T = 0$ limit behave in CeCoIn$_5$ as the field is tuned toward $H_c$. These extrapolations to $T = 0$ of the low-temperature thermal resistivity, defined as $w = \rho_0T/k$, and electrical resistivity $\rho$, for current directions parallel ($J \parallel c$) and perpendicular ($J \perp c$) to the tetragonal axis of the crystal lattice. The raw data and their extrapolation are shown in detail in (4). For $H = 10 T$, far away from $H_c$, $w(T)$ and $p(T)$ converge as $T \to 0$ for both current directions. However, very close to the QCP, for $H = 5.3 T$, they only converge for in-plane transport. In other words, transport along the $c$ axis violates the WF law, with $w_c$ extrapolating to a distinctly lower value than $w$, as the difference data, $w_c(T) - p(T)$ versus $T$, shows a rigid $T$-independent shift from field to field. The normalized Lorenz ratio, $L_c = L/T = p/T$, is also seen to approach unity at 10 T but not at 5.3 T.

Our observation of a violation of the WF law at a QCP is characterized by three distinctive
features: (i) charge conduction is essentially unperturbed; (ii) heat conduction becomes less efficient, so that $L < L_0$; and (iii) the violation is qualitatively anisotropic, present in one direction and absent in the other.

The constancy of charge conduction distinguishes this from the two known instances of WF violation. The first occurs in the case of superconductivity, where $\sigma$ immediately goes to infinity as $H$ drops below $H_{c2}$, whereas $x$ drops gradually, so that $L/L_0 = 0$. The second instance occurs in the other limit ($\sigma \rightarrow 0$), realized in the crossover from a metal to an insulator. In this limit, a violation has been observed in cuprates as the Mott insulating state is approached (4). The case of CeCoIn$_5$ is neither one of superfluid condensation nor one of charge localization, but that of a good metal violating the WF law. The fact that it is a downward violation, $L (T \rightarrow 0) < L_0$, seems inconsistent with the possibility of neutral fermionic excitations such as those predicted to emerge at a heavy-fermion QCP (16). Instead, we will argue that the Fermi surface is destroyed, anisotropically.

In the $T \rightarrow 0$ limit, the WF law holds as long as there is a step in the Fermi distribution function, that is, as long as a sharp Fermi surface exists. This step is proportional to the renormalization parameter $Z$, the defining property of a Landau quasiparticle (17). In standard FL theory, $Z$ is a measure of how strongly the quasiparticle mass $m^*$ is enhanced by electron interactions, with $Z \sim 1/m^*$. The anisotropic violation seen in CeCoIn$_5$ thus suggests that a sharp Fermi surface does not exist in the $c$ direction but does exist in the plane. (This is consistent with the observation of de Haas–van Alphen oscillations in CeCoIn$_5$ for $H \parallel c$ (18), because these result from coherent electron orbits in the plane.) In other words, $Z$ must be a function of polar angle, $Z = Z(\theta)$, whereby $Z \rightarrow 0$ over a region around the “poles” ($c$ axis) and $Z > 0$ in a region around the “equator” (basal plane), which provides evidence of anisotropic zeros in the $Z$ parameter of a metal due to a QCP.

An anisotropic destruction of the Fermi surface is reminiscent of what occurs in the pseudogap state of underdoped high-temperature superconductors, where photoemission studies have revealed a Fermi surface broken into small arcs (19), shrinking to points along “nodal” directions ($\theta = \pi/4$) as $T \rightarrow 0$ (20). This angle-dependent destruction may be caused by strong antiferromagnetic (AF) correlations; it certainly is predominant at points connected by the AF ordering vector. By analogy, the uniaxial destruction of the Fermi surface in CeCoIn$_5$ may be caused by spin fluctuations with a uniaxial character, a scenario which is consistent with both the known fluctuation spectrum and the finite temperature properties discussed below.

Having focused on the $T = 0$ limit, we now examine how quantum criticality unfolds as a function of $T$. The electrical resistivity of CeCoIn$_5$ at the QCP is plotted up to 15 K (Fig. 3) for both current directions. $\rho_c$ shows a purely linear $T$ dependence, from 0.4 $\mu\Omega$ cm at 25 mK all the way to 40 $\mu\Omega$ cm at 16 K. This 100-fold increase in resistivity extends by one order of magnitude the range over which criticality has so far been observed to persist in any material, proving beyond doubt that the power law is an intrinsic property of electrons scattered by critical fluctuations. $\rho_c$ is qualitatively different. Its linear $T$ dependence is seen only above 4 K or so, crossing over to a $T^{3/2}$ dependence below $\approx 1$ K (21).

A comparison of heat and charge conductivities reveals information about the momentum dependence of inelastic scattering. This was discussed in detail in the context of our study of CeRhIn$_5$ (22), the antiferromagnetic cousin of CeCoIn$_5$, with a Néel ordering temperature $T_N = 3.8$ K. The main piece of information that can be extracted directly is the characteristic temperature $T_{SF}$ of magnetic fluctuations, defined as the temperature above which the WF law is restored, i.e., $\nu(T) \approx (T - T_{SF})$. $T_{SF}$ is the magnetic analog of the Debye temperature, the characteristic temperature for the scattering of electrons by phonons, $\Theta_D$.) In CeRhIn$_5$, $T_{SF} \approx 8$ K (22), in good agreement with the onset of AF correlations seen with neutron scattering. In CeCoIn$_5$, the same approach applied to in-plane transport yields a field-dependent $T_{SF} \approx 4$ K at $H_c$ and rising to match that of CeRhIn$_5$ at high field (21). This allows us to understand the strange behavior of $\rho_c$. The linear-$T$ regime above $T_{SF}$ arises from fluctuations without preferred spatial correlations, effectively scattering electrons on the whole Fermi surface and making it uniformly “hot.” (A $T$-linear resistivity is also found in conventional metals when $T > \Theta_D$.) Below $T_{SF}$, the emergence of AF correlations peaked at certain $q$ vectors in the plane leads to “hot spots” and the higher $T^{3/2}$ power law at low $T$. The fact that $\rho_c$ remains linear down to the lowest temperatures suggests that $T_{SF} \approx 0$ in this case, that no interplane correlations build up, and that the Fermi surface remains hot over large regions (away from the plane). [In our phonon analog, this would imply $\Theta_D \rightarrow 0$, a quantum melting of the three-dimensional (3D) solid into stacks of solid sheets separated by nonviscous liquid or gas.]

Whereas the $T = 0$ intercepts are different, the linear $T$ dependence of the $c$-axis electrical resistivity is parallelized by the thermal resistivity (Fig. 3, inset): $\nu(T)$ is perfectly linear down to the lowest $T$. Not only is $\nu_c \sim T$ but also the slope of $\nu_c$ is roughly equal to that of $\rho_c$—a confirmation that $T_{SF}$ indeed vanishes in this direction. This indicates that the usual $(1 - \cos \theta)$ vertex—which makes small-angle scattering ineffective in degrading a charge current—is not working in CeCoIn$_5$. It may be unimportant because inelastic scattering is dominated by large-$q$ processes, as one would expect from AF fluctuations, or it may be inoperative for some special reason, as in the “Kondo breakdown” model (23).

In this instance of anisotropic quantum criticality, given that $Z$ and $T_{SF}$ exhibit the same anisotropy ($Z, T_{SF} \approx 0$ along the $c$ axis and $Z, T_{SF} > 0$ in the basal plane), it is tempting to suggest that (i) a vanishing energy scale, $Z \rightarrow 0$, and a
The WF law violation are all related, and (ii) a good indicator for their joint occurrence is a linear-T resistivity. Returning to our comparison with cuprates, a similar connection between \( \rho \sim T \) and \( Z = 0 \) appears to exist there as well. Indeed, a recent measurement of the (azimuthal) anisotropy of the in-plane scattering rate \( T(\phi) \) in an overdoped cuprate (24) revealed that \( T \sim T(T = 0) \), where the Fermi surface is eventually destroyed (at lower doping), and \( T \sim T^2 \) at \( \phi = \pi/4 \), where it survives.

It is instructive to compare our findings with the properties of other materials and theories of quantum criticality. A T^{-3/2} resistivity is observed in CeIn3 near the pressure-tuned QCP where its AF order vanishes (6). CeIn3 is the cubic parent compound of tetragonal CeRhIn5 and, along with the increase in \( c/a \) ratio, the ordering temperature drops from \( T_S \approx 10 \text{ K} \) in the former to \( T_S \approx 3.8 \text{ K} \) in the latter. However, they still have comparable antiferromagnetic fluctuations that are characteristic of the parent compound. Theoretically, a T^{-3/2} resistivity is expected for AF critical fluctuations in 3D from the so-called quantum spin density wave (SDW) model (17, 25, 26). In this scenario, critical scattering is peaked at "hot spots" connected by the AF wave vectors (25). As \( T \rightarrow 0 \), one would expect the Fermi surface to remain sharp everywhere else, and thus the WF law to prevail, as found here for in-plane currents. A T-linear resistivity is observed at the composition-tuned QCP of CeCu5.9Au0.1 (7) and field-tuned QCP of YbRh2Si2 (12), where AF order is thought to disappear. [In these cases, the power law is linear in both high-symmetry directions (15, 27).] The fact that a linear power is inconsistent with the SDW model for AF fluctuations in 3D prompted the proposal of a 2D version (28) and of an alternate theory, where critical scattering is local in space and therefore present at all wave vectors (29). These scenarios would lead to a more extreme breakdown of FL theory, because the Fermi surface is "hot" not only at certain specific spots but everywhere. It was argued in (8) that the specific heat data on Ge-doped YbRh2Si2, which shows a \( C/T \) that exceeds the \( \log(1/T) \) dependence at low temperature, may be an indication of such enhanced breakdown. In CeCoIn5, the fact that in the direction where \( \rho_T \sim T \) that the XF law is violated is certainly consistent with this picture. Clearly, it would be interesting to test the XF law in YbRh2Si2.

Bringing together our findings for \( T \rightarrow 0 \) and \( T \rightarrow 0 \), a picture of qualitative anisotropy emerges, not present in either the SDW model or the local criticality model, at least in their current forms.

A possible origin for this anisotropic criticality is an anisotropic spin fluctuation spectrum. First, an AF instability is present in all three CeMn3 compounds \((M = \text{Co}, \text{Rh}, \text{Ir})\), as shown by the fact that magnetic ordering can be induced by Cd doping (30). Second, a magnetic field does tune the magnetism. In CeRhIn5, under pressure (which enters in many ways more similar to CeCoIn5, e.g., by developing superconductivity with the same \( T_c \)), a magnetic field stabilizes long-range magnetic order (31, 32). In CeCoIn5, it is the magnetic fluctuations that are tuned by a magnetic field (31), with \( T_S \) starting at a value equivalent to that of CeRhIn5 at high fields and then lowered to a minimum at \( H_c \). Third, the AF fluctuations in CeCoIn5 have strongly anisotropic character (33), with magnetic moments well coupled in-plane but weakly coupled interplane. This is consistent with the helical ordering of moments in CeRhIn5, commensurate in-plane and incommensurate along the c axis. Therefore, it seems natural to link this uniaxial anisotropy with the observed anisotropy in \( T_S \), power laws, and \( Z(0) \). What is not yet known is whether a scenario of AF critical fluctuations can indeed cause a violation of the WF law at \( T \rightarrow 0 \).

However, the AF scenario is not the only candidate for the anisotropic quantum criticality of CeCoIn5. The "Kondo breakdown" model proposed recently (23, 34), a type of deconfined QCP where the hybridization between conduction and f electrons goes to zero, captures some of the key signatures — absence of magnetic order in the phase diagram, strong anisotropy, multiple energy scales, and a T-linear behavior of both charge and heat resistivities. Proximity to a Pomeranchuk instability of the Fermi surface can also cause anisotropy in electronic liquids (7). Recent calculations show that the transport decay rate at such a QCP has a linear T dependence everywhere on the Fermi surface except at "cold" points, resulting in a T^{3/2} dependence of the resistivity (35).

Fig. 3. Anisotropic quantum criticality. Electrical resistivity at the QCP (at \( H = 5.3 \text{ T} \), \( T \approx H \)) for in-plane (\( \rho_T \)) and inter-plane (\( \rho_x \)) current directions. \( \rho_T \) (T) remains linear over a 100-fold increase in magnitude. By contrast, \( \rho_x \) is linear only above a characteristic fluctuation temperature \( T_F \approx 4 \text{ K} \) (arrow) (18). Inset] Thermal resistivity (\( \rho_T \approx L_T \)) at the QCP, for inter-plane transport. \( \rho_T \) is perfectly linear down to the lowest temperature.

References and Notes

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4. Additional data, analysis, and discussion are available as supporting material on Science Online.
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