Response to Comment on "Nonreciprocal Light Propagation in a Silicon Photonic Circuit"

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Fan *et al*. raised technical concerns about our study regarding the Lorentz reciprocity theorem, with which we completely agree. Unfortunately, we incorrectly used the term "nonreciprocal" to describe the behavior of electromagnetic propagation in our devices. In our paper, this term does not refer to Lorentz reciprocity but to the asymmetric mode conversion that is experimentally demonstrated.

We recently demonstrated a silicon (Si) waveguide-based device that performs mode conversion in only one direction (*1*). We called this effect nonreciprocal, a term that could be misleading as it implies that our device avoids Lorentz reciprocity. Indeed, either a magnetic or a time-dependent response is needed to convert our device into an optical isolator. An analysis of reciprocity of our asymmetric optical mode converter, obtained by deriving the corresponding scattering matrix, confirms that the mode conversion of our device is not sufficient for optical isolation on its own. Nevertheless, we believe, it may provide a useful element for on-chip optical isolation in Si photonics when combined with nonlinear optical wave mixing. Even though our device is not adequate to provide optical isolation, we have demonstrated spontaneous breaking of parity-time symmetry.

Our device uses a complex variation in the effective dielectric constant to generate an asymmetric wave vector q. Reciprocity can be assessed by deriving and analyzing the corresponding scattering matrix by solving the coupled mode equations in (*1*). It can be shown that at the two ends of the device in the z direction, $z = 0$ and $z = L/2$ ($L = 2\pi/q$ is the period of modulations), the fields are as follows:

$$A_1(\frac{L}{2}) = \exp\left(-\frac{B_1}{\pi}L\right)A_1(0) + iC\exp\left(-\frac{B_1+B_2}{2\pi}L\right) \times \int_0^{L/2}\exp\left(\frac{B_2-B_1}{2\pi}L\left(\exp\left(i\frac{2\pi z}{L}\right)\right)\right)dzA_2(0)$$

$$A_2(\frac{L}{2}) = \exp\left(-\frac{B_2L}{\pi}\right)A_2(0)$$

$$A_{-1}(0) = \exp\left(-\frac{B_1}{\pi}L\right)A_{-1}(\frac{L}{2})$$

$$A_{-2}(0) = \exp\left(-\frac{B_2}{\pi}L\right)A_{-2}(\frac{L}{2}) + iC\exp\left(-\frac{B_1+B_2}{2\pi}L\right) \times \int_0^{L/2}\exp\left(\frac{B_2-B_1}{2\pi}L\left(\exp\left(i\frac{2\pi z}{L}\right)\right)\right) \times dzA_{-1}(\frac{L}{2}) \qquad (1)$$

where A_n corresponds to the field strength and B_n is the mode integral for mode n, with positive or negative n indicating forward or backward propagation, respectively. The coefficients C gives the modal power overlap for modes 1 and 2 (i.e., symmetric and antisymmetric modes, respectively), with $C = C_1 = C_2$. If we treat the first and second modes, for both forward and backward propagation, as a four-port system with the two ends of the device as input and output, the above equations yield the following scattering matrix:

$$\begin{bmatrix} A_{-1}(0) \\ A_1(\frac{L}{2}) \\ A_{-2}(0) \\ A_2(\frac{L}{2}) \end{bmatrix} = S\begin{bmatrix} A_1(0) \\ A_{-1}(\frac{L}{2}) \\ A_2(0) \\ A_{-2}(\frac{L}{2}) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}\begin{bmatrix} A_1(0) \\ A_{-1}(\frac{L}{2}) \\ A_2(0) \\ A_{-2}(\frac{L}{2}) \end{bmatrix} \qquad (2)$$

where

$$S=\begin{bmatrix} 0 & \exp\left(-\frac{B_1}{\pi}L\right) & 0 & 0 \\ \exp\left(-\frac{B_1}{\pi}L\right) & 0 & iC\exp\left(-\frac{B_1+B_2}{2\pi}L\right)\int_0^{L/2}\exp\left(\frac{B_2-B_1}{2\pi}L\left(\exp\left(i\frac{2\pi z}{L}\right)\right)\right)dz & 0 \\ 0 & iC\exp\left(-\frac{B_1+B_2}{2\pi}L\right)\int_0^{L/2}\exp\left(\frac{B_2-B_1}{2\pi}L\left(\exp\left(i\frac{2\pi z}{L}\right)\right)\right)dz & 0 & \exp\left(-\frac{B_2}{\pi}L\right) \\ 0 & 0 & \exp\left(-\frac{B_2}{\pi}L\right) & 0 \end{bmatrix}$$

This scattering matrix S is symmetric, indicating that the system is reciprocal (*2*). Consequently, the transmission of both modes is the same for both directions, which has been shown in fig. S1 in (*1*) and by Fan *et al*. (*2*). Moreover, because $S_{32} \neq S_{41}$, the device demonstrated in (*1*) converts the first mode to the second mode only in one direction. The exponential elements in the matrix show that power decays as light propagates because the device introduces loss due to metals.

Nevertheless, the demonstrated asymmetric mode conversion, combined with the inherent optical nonlinearity of Si, may still be useful to achieve nonreciprocal transmission in a Si waveguide on-chip without integrating other complementary metal-oxide semiconductor–incompatible materials. For example, the antisymmetric mode can be designed to be necessary for the phase matching of the third harmonic generation ($k^{3\omega} = 2k_1^{\omega} + k_2^{\omega}$). Improvement of nonlinear efficiency using a slow-light structure (*3*) and vanishing two-photon absorption in Si (*4*) might be important to achieve distinguishable transmission in different directions (*5*).

In summary, the one-way mode converter is Lorentz reciprocal and on its own cannot be used as the basis of an optical isolator. It is also lossy [7 dB for the power in the symmetric mode in (*1*)] and will require gain to be added in order to be useful. Similar optical modulation approaches using exponential functions were theoretically proposed by Greenberg and Orenstein (*6*, *7*), which were not included in our original reference list. We thank Fan *et al*. (*2*) for clarifying our work and helping to avoid misunderstanding.

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