Response to Comment on "Probing the Ultimate Limit of Fiber-Optic Strain Sensing"

G. Gagliardi,* M. Salza, S. Avino, P. Ferraro, P. De Natale

Cranch and Foster argue that the strain resolution of our fiber resonator sensor is not limited by thermal noise. However, extension of common theoretical models to high-finesse passive resonators at low frequency needs further attention. Our findings are consistent with noise of thermodynamic nature for which no experimental evidence in the infrasonic range was available until recently.

Thermally induced phase fluctuations in optical fibers have long been studied because they set a fundamental limit to strain resolution of sensors as well as frequency stability of fiber lasers (*1*, *2*). In our original paper (*3*), we reported an unprecedented resolution level in strain sensing using a high-finesse fiber Bragg-grating (FBG) resonator interrogated by an optical-frequency-comb (OFC) stabilized diode laser. The OFC minimizes noise contribution from the laser, which is usually dominant in similar systems, particularly in the infrasonic frequency range (*4*, *5*). Taking into account the laser-frequency noise when locked to the OFC, we estimated the ultimate limit of our technique to be 120 f$\varepsilon/\sqrt{\mathrm{Hz}}$ at 2 Hz (*3*). Despite that, the strain resolution was found to be above the expected value.

Cranch and Foster (*6*) compare our strain noise with that predicted by their model with an effective fiber length equal to twice the FBGs distance (analogous to fiber lasers) (*7*). We remark that the definition of effective length given in (*7*) is difficult to match with the popular Wanser's model (*8*). Wanser's theory provides the only analytical expression of thermal phase fluctuations for finite-cladding fibers. Over the past two decades, few experiments have confirmed this model for long-fiber interferometers—and those have been only in the acoustic range—whereas strong deviations were found at low frequency (*9*, *10*). A similar theory was developed for fiber lasers, but a discrepancy with Wanser's expression was pointed out, and the measured low-frequency noise was far from the predicted level as well (*7*). To explain this behavior, a different theoretical model was proposed for fiber lasers, based on the introduction of source terms in the stochastic heat diffusion equation (*11*). However, the extrapolation of the predictions of these models at low frequencies is still questionable. To our knowledge, no theory or experiment has been reported to date for high-finesse passive fiber resonators, which leaves open the question of which theoretical approach better describes the observed noise. In the absence of other models, we used Wanser's formula with a finesse-dependent length to account for the power storage inside the cavity due to multiple reflections. Although the use of this scaling factor is arguable, it leads to a factor 8 in the noise calculation compared with the original Wanser's formulation, and it describes well the observed noise. Finally, we note that the OFC noise-limited strain resolution is about a factor 6 below the measured noise floor, thereby indicating that we were approaching the thermal fluctuation level predicted by Wanser's expression (*8*), even considering the actual length of our fiber sensor. On the other hand, very recently a new model for low-frequency thermally induced length fluctuations in optical fibers was proposed by Duan (*12*, *13*), following an approach based on the fluctuation-dissipation theorem (*14*). A comparison with our data is already shown by Cranch and Foster (*6*), but a deeper analysis reveals that the expression derived from (*12*) is much closer to experimental findings than others. The formula is based on the calculation of the dissipated elastic energy in a bar of finite length. The dissipation is expressed by the structure damping model through a loss angle ϕ, whose value for acrylic-coated silica fibers is only available in the 75- to 200-kHz range (*15*). In Fig. 1, we show the noise spectrum re-calculated from (*12*), using the real part of Young's modulus recently measured in (*16*) and the lowest-frequency ϕ obtained from (*15*). Actually, there is no direct evidence that this value is valid at frequencies below 1 kHz, and it is suggested that, in principle, the loss angle is a function of frequency (*17*). Nonetheless, the resulting theoretical curve resembles our data between 1 Hz and 10 Hz. In Fig. 1, we also show the root mean square (RMS) sum between the comb-induced laser-frequency noise and the thermal noise spectrum derived from (*12*), which agrees well with our data for $f > 1$ Hz. It is interesting to note that the noise spectrum calculated from (*12*) is markedly different from that obtained by Cranch and Foster over the same frequency range. Furthermore, the model developed in (*12*) is the only one that accounts for a noise roll-up at low frequencies in passive fibers.

We fully investigated all known noise sources in (*3*). The locked laser-frequency noise calculated from local oscillator (LO) phase-noise multiplication in the OFC was found to be well below the measured strain noise level. Extra phase noise was injected in the system via the LO as a countercheck, and the resulting strain spectrum demonstrated that the observed detection limit was not due to the residual laser-frequency stability [supporting online material for (*3*)]. Environmental disturbances were minimized by careful thermal, acoustic, and seismic insulation. Shielding from low-frequency vibrations was provided by a long elastic suspension, with resonance around 0.7 Hz: Neither seismic noise ($\propto 1/f^2$) below 1 Hz nor low-pass filtering from the suspension ($\propto 1/f^2$) above 1 Hz were visible in the measured RMS strain spectral densities. In our opinion, all the above sources can be ruled out from the noise

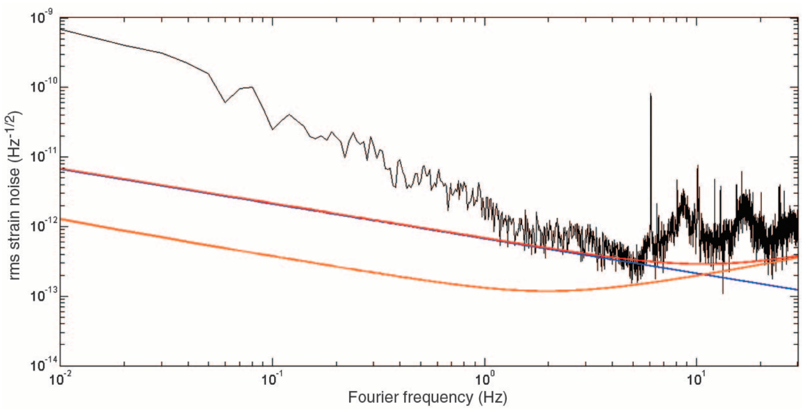

Fig. 1. Strain noise measured by the FBG resonator in (*3*). The blue line is the noise calculated from Duan's model (*12*), the orange line is the comb-stabilized strain noise, and the red line is the RMS sum of them.

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budget. Accordingly, we stand by our conclusion that the measured excess noise can be of thermal origin.

A more general theory of phase noise of optical fiber resonators should consider all thermally induced noise sources. Actually, neither the model in (*12*) nor the more popular theories in (*7*, *8*) considered thermal effects due to light storage in fiber resonators. Photothermal effect caused by the optical loss in photosensitive fibers and Bragg gratings has to be taken into account as a source term for calculating thermal fluctuations in the fiber. That would justify a dependence on the cavity finesse that may influence the overall assessment of thermodynamic noise. Also, thermally excited vibrations in the fiber cavity may add extra noise in the acoustic frequency range (*9*).

Cranch and Foster remark that thermally induced strain noise is very difficult to observe below 100 Hz. We agree that sophisticated laboratory experiments are required to observe these phenomena. However, we note that ours is the first sensor that is capable of detecting sub-picostrain signals in the infrasonic range. We believe that the diverse theories proposed so far to describe thermal noise really need experimental validations to provide a meaningful description of physical systems. For instance, the theory proposed in (*12*) yields a frequency response of the type $1/\sqrt{f}$, and the temperature dependence is $\sqrt{T}$. This is in evident disagreement with other models (*7*, *8*), which predict, for passive fibers, an almost flat frequency response and a temperature dependence on T.

In conclusion, we thank Cranch and Foster for opening with their comment a debate on present models and laboratory experiments. Although a validation of theoretical models was beyond the scope of our original paper as well as of this response, we are confident that our contribution can stimulate novel approaches in this field.

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