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Supporting Online Material for

The Mass of Dwarf Planet Eris

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Materials and methods

Keck LGS AO observations were obtained on 20, 21, 30, and 31 August 2006 UT using NIRC2, the facility near infrared camera, at the astronomical H- and K-wavelength bands centered at 1.5 and 2.0 μm, respectively. Detection of Dysnomia was challenging even for the Keck LGS AO system, however, and the satellite was only marginally visible in the sum of the ~100 60 s exposures from each night. To increase image contrast, the images were sorted by quality of the image correction, and only the top 20% were used. The position of Dysnomia in each image (Table S1) was measured by subtracting a radially-averaged model for Eris from the image and then measuring centroids on the resulting residual image of Dysnomia. Uncertainties were estimated from the standard deviation of individual measurements.

HST observations were obtained on 3 Dec 2005 and 30 August 2006 UT using the HRC/ACS instrument with the F606W filter with total exposure times of 600 and 4460 s, respectively. For these measurements TinyTim point-spread function (PSF) modeling software (*1)* was used to construct a theoretical image of a point source, and the position of Dysnomia (Table S1) was found by performing a least-squares fit to the sum of two such point sources. Uncertainties were again estimated from the standard deviations of multiple measurements.

To determine uncertainties in the derived orbital parameters, we perform 1000 iterations of circular orbit fit optimization where we add gaussian noise with σ equal to the measurement uncertainties of the position measurements, and we solve for new orbital parameters. We define the 1 σ uncertainties on the parameters to be the range containing the central 68% of the data. To estimate an upper limit to the eccentricity, we perform an additional 1000 iterations allowing a fully eccentric fit and take the 1 σ upper limit to eccentricity to be the value higher than 84% of the data. Table 2 gives the ecliptic orbital elements of the two satellite orbits (which appear identical in projection and cannot yet be distinguished) which provide excellent fits to the data.

Assuming that the pole of the rotation axis of Eris and the pole of the orbital plane of Dysnomia are coincident, as they should be for a tidally evolved circular orbit with no additional perturbations, we can use the orbit of Dysnomia to determine the obliquity, sub-solar latitude, and other seasonal parameters of Eris. These parameters derived from the orbit of Dysnomia are shown in Table S2.

Supporting Text

We can estimate the expected orbital period of a satellite after 4.5 billion years of tidal evolution from equation 4.214 of Murray and Dermot (*2*,*3*) as

$$P = 15\ \text{days}\left[\frac{k}{1.5}\frac{100}{Q}\frac{7300}{q}\right]^{3/13}\left[\frac{\rho}{2.3\ \text{g/cm}^3}\right]^{-5/13},$$

where k is tidal Love number, Q is the quality factor of the primary, q is the ratio of the primary to satellite mass, which we estimate to be of order 7300 assuming that the satellite has a similar albedo and half the density of the primary, and ρ is the density of the primary (which we measure below to be 2.3 g cm^{-3}). For reasonable values of these parameters, this estimate for the expected orbital period from tidal evolution is in excellent agreement with the measured period of nearly 16 days.

Tidal evolution will affect the eccentricity as well as the period. The timescale for eccentricity damping relative to orbital expansion can be estimated from equation 4.198 of Murray and Dermot (*2,3*) as

$$\left|\frac{\dot{e}/e}{\dot{a}/a}\right| = \frac{7}{2}\left(\frac{m_p}{m_s}\right)\left(\frac{r_s}{r_p}\right)^5\left(\frac{k_s}{k_p}\right)\left(\frac{Q_p}{Q_s}\right) = \frac{7}{2}\left(\frac{r_p}{r_s}\right),$$

where *m* is the mass and *r* is the radius of the body and the subscripts *r* and *p* refer to the satellite and primary, respectively. The second inequality assumes that both bodies have similar quality factors and similar densities. The eccentricity damping timescale is about 80 times faster than the orbital evolution timescale, which, by assumption is the age of the solar system. Therefore, the eccentricity of the satellite damps every 50 Myr and should be expected to be extremely small, in agreement with the observations.

We use the deep HST image from 30 August 2006 to explore the possibility of additional satellites in the Eris system. To examine the possibility of satellites more distant than Dysnomia we add together all data from two orbits of HST observations and place artificial images of satellites into the scene to determine our detection limits. We find that outside of the orbit of Dysnomia we can rule out the existence of any additional satellites to a brightness level of 0.0005 the brightness of Eris (almost an order of magnitude fainter than Dysnomia).

Interior to Dysnomia the detection of faint satellites is made difficult by the additional light from the wings of the point-spread-function (PSF) of Eris itself. We use the Tiny Tim PSF modeling software [tinytim] to construct theoretical models of the PSF of the HST, convolve these with the known angular size of Eris, and subtract these models from the individual mages. For each image, the best-fit PSF is found by minimizing the square of the residuals between the model and the image while allowing the first eight Zernike terms, which describe the dominant modes of the aberration of the HST, to vary. After subtracting the model from each image we added the individual images together and again placed artificial satellites into the scene to determine our detection limits. Owing to the steep gradient in the PSF, the limits vary greatly with distance from Eris. Within 0.1 arcseconds of Eris we could only have detected a satellite with a brightness within an order of magnitude of that of Eris or higher. At distances of 0.12, 0.20, 0.25, 0.40, and 0.45 arcseconds from Eris we could have detected satellites with fractional brightnesses of 0.05, 0.01, 0.005, 0.002, and 0.0007, respectively.

Assuming that any additional satellites have tidally evolved similarly to Dysnomia, their brightness can be used to estimate a mass and thus an orbital period and semimajor axis from the above equation.

Supporting Tables

Table S1: Separation of Dysnomia from Eris

Julian date	Telescope	R.A. Offset (mas)	Dec. Offset (mas)
2453624.02	Keck	-520 ± 8	80 ± 8
2453707.94	HST	206 ± 6	-338 ± 6
2453968.12	Keck	-148 ± 10	348 ± 10
2453969.09	Keck	-360 ± 40	260 ± 40
2453978.08	Keck	488 ± 20	-200 ± 30
2453978.37	HST	504 ± 1	-167 ± 1
2453979.00	Keck	520 ± 10	-72 ± 10

Table S2: Parameters of the orbit of Dysnomia

Orbital parameters	Orbit 1	Orbit 2
Semimajor axis......................	37430±140 km	37370±150 km
Inclination...............................	61.3±0.7 degrees	142±3 degrees
Period....................................	15.772±0.002 days	15.774±0.002 days
Eccentricity............................	<0.010	<0.013
Longitude of ascending node.	139±1 degrees	68±3 degrees
Mean anomaly........................	328.6±0.6 degrees	306.5±1.3 degrees
Epoch (defined)......................	2453979.00	

Derived parameters for the spin of Eris		
Obliquity	78 degrees	
Current sub-solar latitude	39 degrees	
Year of vernal equinox	AD 2239.5	AD 2126.5
E_m	339.3 degrees	251.3 degrees

NOTES. – We define E_m as the angle between the perihelion of the orbit of Eris and the longitude of the descending node of the relative orbit of Dysnomia, or equivalently, the eccentric anomaly of Eris at the moment of its vernal equinox. All values are relative to the J2000 ecliptic.

Supporting References

1. J.E. Krist, P. Hook, *The Tiny Tim User;s Guide, version 6.0* (STScI, Baltimore, 2001).
2. C.D. Murray, S.F. Dermott, *Solar System Dynamics* (Cambridge University Press, New York, 1999).
3. M.E. Brown, *et al.*, *Astrophys. J.*, 632, L45-47 (2005).